Mid-Term Examination Spring 1996

Please solve two problems

1. (a) Determine $H_q(S^1 \times S^1)$. Let $p_1, p_2 : S^1 \times S^1 \to S^1$ be the two projections. Find the morphisms $p_{1*}, p_{2*} : H_q(S^1 \times S^1) \to H_q(S^1)$.

   (b) Let m, n be two integers and define $\phi : S^1 \to S^1 \times S^1$ by the formula $\phi(z) = (z^n, z^m)$. Determine ϕ_*.

   (c) Prove that $\tau_* = -1$ on $H_2(S^1 \times S^1)$ if $\tau(z, w) = (w, z)$.

2. For any integer $k \in \mathbb{Z}$ we define $Z_k = D^2/\equiv$, where $D^2 = \{z \in \mathbb{C}, |z| \leq 1\}$, and $z \equiv w$ if and only if $z = w$ or $z^k = w^k$ and $|z| = 1$. Fix two integers m and n.

   (a) Prove that the spaces $X = Z_m$ and $Y = Z_m \times Z_n$ are CW-complexes by providing explicit realizations. (i.e. find the k-skeleta $X_0 \subset X_1 \subset X_2 = X$ and $Y_0 \subset Y_1 \subset \ldots \subset Y_4 = Y$ and describe the cells that are attached and the attaching maps.)

   (b) Determine the chain complexes $(C_n(X), d)$ and $(C_n(Y), d)$ associated to the above CW-complexes.

   (c) Compute the groups $H_q(X)$ and $H_q(Y)$.

3. (a) Determine the morphisms $j_* : H_q(\mathbb{C}P^n) \to H_q(\mathbb{C}P^{n+1})$ induced by the inclusion $j : \mathbb{C}P^n \to \mathbb{C}P^{n+1}$.

   (b) Compute the homology groups $H_q(\mathbb{C}P^{n+k}, \mathbb{C}P^n)$ (try first $k = 1, 2$).

   (c) Determine the morphism $l_* : H_q(\mathbb{R}P^n) \to H_q(\mathbb{C}P^n)$ determined by the natural inclusion, $l : \mathbb{R}P^n \to \mathbb{C}P^n$,

$$l([x_0, x_1, \ldots, x_n]) = [x_0, x_1, \ldots, x_n].$$