MATH 528 FINAL
Solve two problems from each section.
Do not forget to write your name and SS number on your work.

You may get additional points if you solve more problems. Do not panic, however, if you solve less: it still may happen that you get the perfect score even if you do less than two problems from each section. Please make sure that you clearly explain the logic of your solution and justify all statements you make. You may receive partial credit only if you make *essencial* progress towards a solution. I gave hints to a couple of problems. These problems, however, have many different solutions, and it is quite possible that you find easier another solution, which does not use my hint. A topic which each particular problem is supposed to test is given right after its number. This information is also can be regarded as hints. Keep in mind, however, that these problems may also be solved by means of methods different from those which I indicated.
GOOD LUCK!

Section 1. Algebraic topology.

1. (CW-Homology and Universal Coefficients.) To formulate this problem, we represent a 2-sphere S^2 as $S^2 = \{(z,t) \in C \times R : |z|^2 + t^2 = 1\}$, and a 3-ball B as $B = \{(z,t) \in C \times R : |z|^2 + t^2 \leq 1\}$. Let P be a space obtained from $T = S^2 \times S^2$ by attaching a ball B along the map $\phi : S^2 \to T : \phi(z,t) = ((z^4, t), (0, 1))$. Compute the homology groups of P with coefficients in Z, R and Z_6. (Hint: a problem with long formulation may be quite easy...)

2. (Kunneth Formula.) Let K be the quotient space of a 2-disc obtained by identifying points on its boundary which are $\frac{2}{3}\pi$ apart. Find $H_2(K \times K)$ and describe a set of its generators.

3. (Lefschetz Fixed Point Theorem.) Let f, g and h be three homogeneous polynomials in three variables of the same degree. Assume that neither of these polynomials vanishes except at $(0,0,0)$. Prove that the equation $\frac{f(x,y,z)}{x} = \frac{g(x,y,z)}{y} = \frac{h(x,y,z)}{z}$ has a solution. (Hint: what is the compact manifold where it is "natural" to consider this equation?)

Section 2. Differential Topology.

4. (Orientation.) Prove that the tangent bundle of any manifold (not necessarily orientable) is an **orientable** manifold.

5. (Gradient Flow.) M is a smooth compact connected manifold and $f : M \to R$ is a smooth function with finitely many critical points. Prove that, if the num-

ber of critical points is not equal to 2, the f has a critical point which is neither a local maximum nor a local minimum. (Hint: you may look at gradient curves of f and ask where they may "begin" and "end".)

6. (De Rham's Theorem.) M is a manifold covered by S^n. ω is a closed 1-form on M. Prove that ω is exact.