—————— Math 527— Final————GOOOD LUCK—————————

One completely solved problem from each section problems guarantees you the perfect score.
Please make sure that you justify all statements you make!
Good luck!

Section 1.

1. Is it true that every distance-decreasing map $f : Y \to L_\infty(X)$ from a closed subset Y of a compact metric space X extends to a distance-decreasing map $f_1 : X \to L_\infty(X)$? (That is, f is the restriction of f_1 onto Y. The distance on $L_\infty(X)$ is defined, as usual, as dist(f, g)=$\sup_{x\in X} |f(x) - g(x)|$.)

2. Do there exist two non-homeomorphic spaces X and Y such that X contains a subset homeomorphic to Y a nd Y contains a subset homeomorphic to X?

Section 2.

3. Does there exist an open set in some Euclidean space with **finite non-trivial** fundamental group?

4. Consider the sphere $S^2 = \{(x, y, z) \in R^3, x^2 + y^2 + z^2 = 1.\}$, with the equivalence relation $(x, y, z)\mathbf{E}(p, q, r)$ iff $|z| = |r| = 1/2, x = p, y = q$. Find the fundamental group of the quotient space $S^2/\mathbf{E}$.

Section 3.

5. Let K be the two-dimensional skeleton of the standard 4-dimensional simplex. Find the homology groups $H_n(K)$.

6. Let $S^4 = \{(x_1, x_2, x_3, x_4, x_5) \in R^5 : \Sigma x_i^2 = 1)\}$, and $M \subset S^4 = \{(x_1, x_2, x_3, x_4, x_5) \in R^5 : x_1^2 = x_2^2 + x_3^2 = x_4^2 + x_5^2 = 1/3\}$. Find the relative groups $H_n(S^4, M)$.