MATH 536 Midterm, Spring 1997

Do any four problems, but not both **1.** *and* **2.**

1. Determine the rational canonical form for the matrix: $\begin{pmatrix} 1 & -1 & -4 \\ 0 & 0 & 3 \\ -1 & 0 & 2 \end{pmatrix}$.

2. Classify $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$ over a field F up to similarity in the two cases:

(a) *char* $F \neq 2$. **(b)** *char* $F = 2$.

3. Let $f(X)$ be an irreducible polynomial over $\mathbf{Q}$ with roots $\alpha_1, ..., \alpha_n$ in $\mathbf{C}$. Show that if $i \neq j$, then $\alpha_i - \alpha_j \notin \mathbf{Q}$.

4. Let E be the splitting field of $X^{15} - 1$ over the rational field $\mathbf{Q}$.

(a) Find the galois group of E over $\mathbf{Q}$.

(b) Find all subfields between $\mathbf{Q}$ and E, and give generators for each.

5. Prove that the group $GL(2, \mathbf{F}_3)$ is solvable.

6. Let $E = \mathbf{F}_3(t)$ where t is transcendental over $\mathbf{F}_3$. Let G be the subgroup of Aut E generated by σ, where $\sigma(t) = 2t + 1$, and let $F = \text{Inv}(G)$ be the subfield of E fixed by G. Find:

(a) $[E : F]$,

(b) the minimum polynomial of t over F.