Math 140
Sample Exam

The use of books, notes, and calculators is not permitted on this exam.

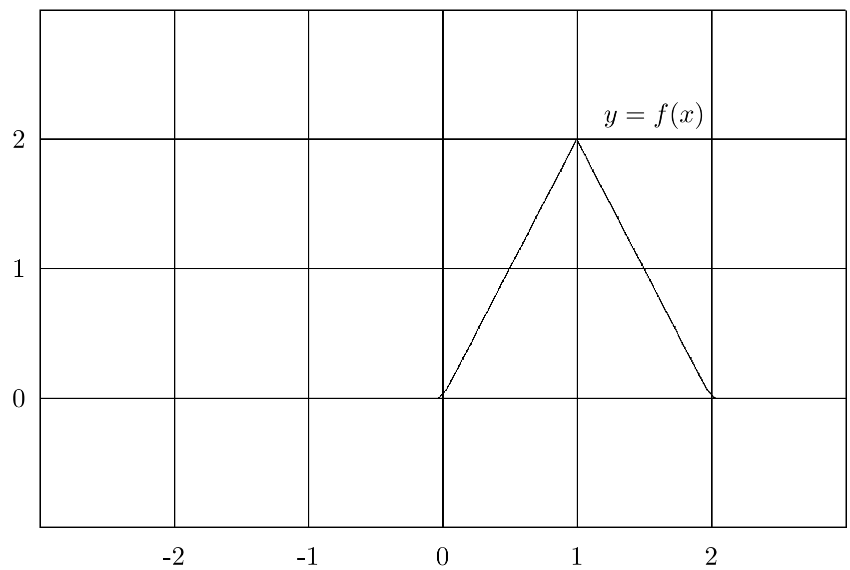
1. **a.** Solve the following inequality and express the solution set as a union of intervals.

$$|x - 1| \geq 3$$

b. Find the equation of the line that passes through the points $A(1, -2)$ $B(5, 6)$. Also find the y-intercept of this line.

c. Consider the graph of the function $y = f(x)$ given below. Sketch the graph of the function

$$y = \frac{1}{2} f(x + 1)$$



2. **a.** A car takes 20 seconds to accelerate from a standing start to 90 mph, and it requires 1/3 of a mile to do this. What is the average velocity of the car over this interval of time?

b. Find the limit $\lim_{x \to 0} \dfrac{x^2 + 21x}{x}$.

c. Find the limit $\lim_{x \to 0} \dfrac{\sqrt{x + 81} - 9}{x}$.

3. **a.** Find the limit $\lim_{x \to 1^+} \dfrac{|1 - x|}{1 - x}$.

b. Find the value of the number a such that the function

$$f(x) = \begin{cases} x + a & \text{if} \quad x \leq 3 \\ \dfrac{x^2 - 9}{x - 3} & \text{if} \quad x > 3 \end{cases}$$

is continuous everywhere.

c. Find the limit $\lim_{x \to 0} \frac{\sin 3x}{x}$.

4. Use the **definition** of derivative to find the derivative of the function $f(x) = \frac{1}{x}$ at $x = 2$.

5. **a.** Find the first derivative and second derivative of

$$\frac{x^2 + 1}{x^2}$$

b. At time $t \geq 0$ hours, the velocity of a car travelling down a straight highway is $v(t) = 25(16t - t^2)$ mph. Find when the velocity of the car is decreasing, and find when the car is being driven backwards.

6. **a.** Find the derivative of $y = x \tan x$.

b. Find the derivative of $y = \frac{\cos x}{1 - \sin x}$.

7. **a.** Consider the table of values for two functions and their derivatives $f(x)$, $g(x)$ at $x = 1$, $x = 2$, $x = 3$. Find $(f \circ g)'(1)$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$.
1	3	2	8	9
2	1	3	6	7
3	2	1	4	5

b. Find the derivative of the function $y = (x + x^2)^{1/3}$.

c. Find the derivative of the function $y = \cos(\sqrt{2x + 1})$.

8. **a.** Find the derivative of the function $y = \sqrt{x^3}$.

b. Find the derivative of the function $y = x^{1/2}(x^2 + x)$.

c. Assume that the equation $x^2 + xy + y^3 = 10$ defines y as a function of x near the point $(1, 2)$. Find the value of the derivative $\frac{dy}{dx}$ at that point.

9. A 6-foot-tall man is walking away from a street light which is 18 feet tall. Assume that the man's speed is 3 feet/sec and assume that the light casts a shadow on the ground. How fast is the length of his shadow increasing?

10. **a.** Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 9x^2 + 12x + 1$ on $[0, 3/2]$.

    b. Sketch a graph of the above function on $(-infty, infty)$ and identify the points on the graph where the absolute extrema occur.

11. **a.** For each one of the following functions and intervals, state why the the hypotheses of the Mean Values Theorem are **NOT** satisfied.

    i. $f(x) = |x - 1|$ on $[0, 2]$

    ii. $f(x) = \sec x$ on $[0, 2]$

    b. Find a value of c that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^2 + 2x$ on the interval $[1, 3]$.

    c. Use Rolle's theorem to explain why $x^2 - 2$ can have only one root between 1 and 2.

12. **a.** Determine the intervals on which the function $f(x) = x^{5/3} - 10x^{2/3}$ is increasing and the intervals on which it is decreasing.

    b. Determine the intervals on which the function $f(x) = 6x^5 - 5x^6$ is increasing and the intervals on which it is decreasing.

13. Use all the calculus techniques at your disposal to sketch the graph of the following function

$$f(x) = \frac{x^2 + 4}{x}.$$

    Specifically, use symmetry, dominant term, asymptotes, critical points and intervals of increase or decrease, concavity and inflection points, as appropriate.

14. A rectangular plot of land has one of its sides along a river, and on the other three sides, a fence. What is the the largest area that can be enclosed using 100m of fencing material? Solve this problem using the following steps.

    a. Express A area as a function of a single variable. Find a closed interval I on which you are certain that A assumes its maximum.

    b. Find the critical point(s) of A.

    c. Find the absolute maximum of A on I.

15. Use differentials to approximate the volume of a cube whose edge is 2.1 cm.

16. Suppose you use Newton's method to find a zero of the function
    $f(x) = x^2 - 3x + 3/2$.

a. Explain why this function must have a zero between 1 and 3.

b. What number is a bad choice for the initial guess in Newton's method?

c. Make a reasonable initial guess at a root, and apply Newton's once method to improve that approximation.

17. **a.** Find

$$\int x^2 + x^{-2}\, dx.$$

b. Find $\int \frac{1}{2} \cos 2x\, dx$.

c. Find $\int x^{-1/2}\, dx$.

d. Find $\int \sec 2x \tan 2x\, dx$.

18. Suppose that the velocity v of an object which moves along a horizontal line at time t is given by $\sin 2t$. Also assume that its position $s(0)$ at time $t = 0$ is 0. Find the position of the object at any time $t > 0$.

19. Evaluate the following indefinite integrals

a. $\int \sqrt{2 + 3x}\, dx$

b. $\int \cos^2 x \sin x\, dx$

20. **a.** Write the following sum without sigma notation

$$\sum_{i=2}^{5} (i+1)^2 - i^2$$

b. Express the following sum using sigma notation

$$\frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7}$$

c. Let

$$S_n = \frac{2}{n}\left[\frac{6}{n} + \frac{12}{n} + \frac{18}{n} + \ldots + \frac{6(n-1)}{n} + 6\right]$$

Calculate $\lim_{n\to\infty} S_n$ by recognizing that S_n is an approximation of the integral

$$\int_0^b mx\, dx$$

for suitably chosen constants b and m, and then evaluating the integral.

21. **a.** Find the area of region bounded by the graph of the function $y = x^2 - x$ and the x-axis.

    b. Evaluate the integral

$$\int_0^{\pi/2} 1 - \sin x \, dx$$

22. **a.** Using the shift formula (i.e., the substitution $u = x + c$), express the following integral

$$\int_2^3 \frac{1}{x+5} \, dx$$

as an integral over the interval $[0, 1]$. (**DO NOT** try to evaluate the integral.)

    b. Evaluate the following integral

$$\int_0^4 x\sqrt{x^2+1} \, dx$$

23. Using the following table of values for a function $f(x)$, estimate the integral $\int_1^3 f(x)\, dx$ using the trapezoidal rule with $n = 4$.

x	**f(x)**
0.00	-1.00
0.25	0.00
0.50	1.00
0.75	2.00
1.00	3.00
1.25	4.00
1.50	3.00
1.75	2.00
2.0	1.00
2.25	0.00
2.50	-1.00
2.75	0.00
3.00	1.00
3.25	2.00

24. Find the area enclosed by the curves with the equations $x = y^2$, $y = x - 2$. (Start by sketching the curves.)

25. Find the volume of the solid generated by revolving the region between the graphs of $y = 4x - x^2$ and $y = 0$ about the x-axis. (Start by sketching the curves.)

26. Find the volume of the solid generated by revolving the region between the graphs of $y = x - x^2 + 2$ and $y = 2$ about the x-axis. (Start by sketching the curves.)

27. Find the volume of the solid generated by revolving the region between the graphs of $y = x - x^2$ and $y = 0$ about the y-axis. (Start by sketching the curves.)

28. Find the length of the curve given by the equation $y^3 = x^2$ joining the points $(0, 0)$ and $(1, 1)$.

29. Two straight highways intersect at point O. One highway runs north-south and the other runs east-west. At 12 noon, a car on the east-west highway is 3 miles west of the intersection O going east at the rate of 60 miles an hour. At the same time, another car on the north-south highway is 4 miles north of the intersection travelling north at the rate 70 miles an hour. At this time, what is the rate of change of the distance between the cars?