Section 8.4 Circles

Objective 1: Find the Distance between Two Points

Recall that the Pythagorean Theorem states that the sum of the squares of the two sides of a right triangle is equal to the square of the hypotenuse or \( a^2 + b^2 = c^2 \).

The Pythagorean Theorem \( a^2 + b^2 = c^2 \)

We can use the Pythagorean Theorem to find the distance between any two points in a plane.

The Distance Formula

The distance between any two points \( A(x_1, y_1) \) and \( B(x_2, y_2) \) is given by the formula

\[
d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

8.4.7 Find the distance \( d(A, B) \) between points \( A \) and \( B \). Simplify your answer. Type an exact answer, using radicals as needed.

Objective 2: Find the Midpoint of a Line Segment

The midpoint of the line segment from \( A(x_1, y_1) \) to \( B(x_2, y_2) \) is

\[
M(x, y) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

Midpoint Formula

The midpoint of the line segment from \( A(x_1, y_1) \) to \( B(x_2, y_2) \) is

\[
M(x, y) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).
\]
8.4.16
Find the midpoint of the line segment joining points A and B. Type an ordered pair. Simplify your answer.

**Objective 3: Write the standard form of an equation of a circle**

A **circle** is the set of all points \((x, y)\) in the Cartesian plane that are a fixed distance \(r\) from a fixed point \((h,k)\). The fixed distance \(r\) is called the **radius** of the circle and the fixed point \((h,k)\) is called the **center** of the circle.

A circle with center \((h,k)\) and radius \(r\)

![Diagram](image)

**The standard form of an equation of a circle** with center \((h,k)\) and radius \(r\) is

\[
(x-h)^2 + (y-k)^2 = r^2.
\]

The standard form of an equation of a circle centered at the origin with radius \(r\) \(x^2 + y^2 = r^2\).

8.4.21 Write the standard form of the equation of the circle described.
Objective 4: Sketch the graph of a circle given in standard form

Once we know the center and radius of a circle, we can easily graph the circle. For additional points, find any intercepts and plot the points.

Note that the y-coordinate of the center of the circle $(x - 1)^2 + (y + 2)^2 = 9 (k = -2)$ is negative because $(y + 2)^2 = (y - (-2))^2$.

8.4.26 Find the center and radius of the circle and sketch its graph.

Objective 5: Write the general form of a circle in standard form and sketch its graph

The general form of the equation of a circle is $Ax^2 + By^2 + Cx + Dy + E = 0$ where $A, B, C, D,$ and $E$ are real numbers and $A = B$.

By completing the square, the equation of a circle can be rewritten from general form to standard form.

8.4.36 Find the center, radius, and intercepts of the circle with the given equation and then sketch the graph.