Section 7.3 Rational Exponents and Simplifying Radical Expressions

Objective 1: Use the Definition for Rational Exponents of the Form \( a^{1/n} \)

### Definition
**Rational Exponent of the Form** \( a^{1/n} \)

If \( n \) is an integer such that \( n \geq 2 \) and if \( \sqrt[n]{a} \) is a real number, then \( a^{1/n} = \sqrt[n]{a} \).

7.3.4 Write the exponential expression as a radical and simplify, if possible.

Objective 2: Use the Definition for Rational Exponents of the Form \( a^{m/n} \)

### Definition
**Rational Exponent of the Form** \( a^{m/n} \)

If \( \frac{m}{n} \) is a rational number in lowest terms, \( m \) and \( n \) are integers such that \( n \geq 2 \), and \( \sqrt[n]{a} \) is a real number, then \( a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m} \).

7.3.9 Write the following exponential expression as a radical and simplify, if possible.

Objective 3: Simplify Exponential Expressions involving Rational Exponents

### Rules for Exponents

- **Product Rule**  
  \( a^m \cdot a^n = a^{m+n} \)

- **Quotient Rule**  
  \( \frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0) \)

- **Zero-Power Rule**  
  \( a^0 = 1 \quad (a \neq 0) \)

- **Negative-Power Rule**  
  \( a^{-n} = \frac{1}{a^n} \) or \( \frac{1}{a^m} = a^{-n} \quad (a \neq 0) \)

- **Power-to-Power Rule**  
  \( (a^m)^n = a^{mn} \)

- **Product-to-Power Rule**  
  \( (ab)^n = a^n b^n \)

- **Quotient-to-Power Rule**  
  \( \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \quad (b \neq 0) \)
Recall that an exponential expression is simplified when:

- No parentheses or grouping symbols are present.
- No zero or negative exponents are present.
- No powers are raised to powers.
- Each base occurs only once.

7.3.21 Write the following exponential expression with positive exponents. Simplify if possible.

7.3.23 Use the rules for exponents to simplify the following expression.

**Objective 4: Use Rational Exponents to Simplify Radical Expressions**

<table>
<thead>
<tr>
<th>Using Rational Exponents to Simplify Radical Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1.</strong> Convert each radical expression to an exponential expression with rational exponents.</td>
</tr>
<tr>
<td><strong>Step 2.</strong> Simplify by writing fractions in lowest terms or using the rules of exponents, as necessary.</td>
</tr>
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<td><strong>Step 3.</strong> Convert any remaining rational exponents back to a radical expression.</td>
</tr>
</tbody>
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7.3.37 Multiply and simplify. Assume that all variables represent non-negative values.

**Objective 5: Simplify Radical Expressions Using the Product Rule**

**Product Rule for Radicals**

If \( \sqrt[n]{a} \) and \( \sqrt[n]{b} \) are real numbers, then \( \sqrt[n]{a \cdot b} = \sqrt[n]{ab} \).

**Caution:** The index on each radical must be the same in order to use the product rule for radicals.
Using the Product Rule to Simplify Radical Expressions of the Form $\sqrt[n]{a}$

**Step 1.** Write the radicand as a product of two factors, one being the largest possible perfect $n$th power.

**Step 2.** Use the product rule for radicals to take the $n$th root of each factor.

**Step 3.** Simplify the $n$th root of the perfect $n$th power.

7.3.41 Use the product rule to simplify. Assume all variables represent non-negative values.

7.3.47 Multiply and simplify. Assume all variables represent non-negative values.

**Objective 6: Simplify Radical Expressions Using the Quotient Rule**

**Quotient Rule for Radicals**

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $b \neq 0$, then $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$.

7.3.61 Use the quotient rule to simplify. Assume all variables represent non-negative values.

**Simplified Radical Expression**

For a radical expression to be simplified, it must meet the following three conditions:

**Condition 1.** The radicand has no factor that is a perfect power of the index of the radical.

**Condition 2.** The radicand contains no fractions or negative exponents.

**Condition 3.** No denominator contains a radical.