Section 3.2 Systems of Linear Equations in Three Variables

**Definition Linear Equation in Three Variables**

A linear equation in three variables is an equation that can be written in the form \( Ax + By + Cz = D \), where \( A, B, C, \) and \( D \) are real numbers, and \( A, B, \) and \( C \) are not all equal to 0.

Following is an example of a system of three linear equations in three variables.

\[
\begin{align*}
2x + 3y + 4z &= 12 \\
x - 2y + 3z &= 0 \\
-x + y - 2z &= -1
\end{align*}
\]

The solution to this linear system is the set of all ordered triples \((x, y, z)\) that satisfy all three equations. Graphically, we can view the solutions of a system of three equations in three variables as intersection points of planes.

**Objective 1:** Determine if an Ordered Triple is a Solution to a System of Linear Equations in Three Variables

3.2.2 Two ordered triples are given. Determine if each ordered triple is a solution of the given system.
Objective 2: Solve Systems of Linear Equations in Three Variables

Guidelines for Solving a System of Linear Equations in Three Variables by Elimination

Step 1: Write each equation in standard form
Write each equation in the form $Ax + By + Cz = D$, lining up the variable terms. Number the equations to keep track of them.

Step 2: Eliminate a variable from one pair of equations
Use the elimination method to eliminate a variable from any two of the original three equations, leaving one equation in two variables.

Step 3: Eliminate the same variable again
Use a different pair of the original equations and eliminate the same variable again, leaving one equation in two variables.

Step 4: Solve the system of linear equations in two variables
Use the resulting equations from steps 2 and 3 to create and solve the corresponding system of linear equations in two variables by substitution or elimination.

Step 5: Use back substitution to find the value of the third variable
Substitute the results from step 4 into any of the original equations to find the value of the remaining variable.

Step 6: Check the solution
Check the proposed solution in each equation of the system and write the solution set.

3.2.3 Solve the system of linear equations.
Objective 3: Use Systems of Linear Equations in Three Variables to Solve Application Problems

Extend the problem-solving strategy from Section 3.1 to setting up and solving applications that lead to a system of three equations in three variables.

3.2.19 A concession sells hot dogs, hamburgers, and chicken sandwiches. During game 1 the stand sold _____ hotdogs, _______ hamburgers, and _____ chicken sandwiches for a total of $_______. During game 2, the stand sold _____ hotdogs, _______ hamburgers, and _____ chicken sandwiches for a total of $_______. During game 3, the stand sold _____ hotdogs, _______ hamburgers, and _____ chicken sandwiches for a total of $_______. Determine the price of each product.