Math 568 Homework 9
Spring 2009
Due: Thursday, March 26

1. Marcus, Exercise 3b, page 114: Complete the proof of the quadratic reciprocity law. (You may assume part (a).)

2. Marcus, Exercise 7, page 116: Find a prime p and quadratic extensions K and L of $\mathbb{Q}$ illustrating **2** of the following (pick any 2 you want from the 4 choices below):

   (a) p can be totally ramified in K and L without being totally ramified in KL.

   (b) K and L can each contain unique primes lying over p while KL does not.

   (c) p can be inert in K and L without being inert in KL.

   (d) The residue field extension of $\mathbb{F}_p$ can be trivial for K and L without being trivial for KL.

3. Marcus, Exercise 18, page 121: Let L/K be a Galois extension of number fields, with $G, \mathfrak{p}, Q, D$, and E as usual. Define the ***ramification groups*** for $m \geq 0$:

$$V_m := \{\sigma \in G : \ \sigma(\alpha) \equiv \alpha \pmod{Q^{m+1}} \ \ \forall \alpha \in \mathcal{O}_L\}.$$

   Thus $V_0 = E$, and the V_m form a descending chain of subgroups.

   (a) Prove that each V_m is a normal subgroup of D.

   (b) Prove that the intersection of all V_m is $\{1\}$, hence $V_m = \{1\}$ for all sufficiently large m.