

**Math 568 Homework 7**  
**Spring 2009**  
**Due: Thursday, March 5**

1. Show that  $\mathbb{Z}[\sqrt{-5}]$  has exactly two ideal classes, the class of principal ideals and the class containing  $(2, 1 + \sqrt{-5})$ .
2. Let  $m \neq 1$  be a square-free integer. Let  $K := \mathbb{Q}(\sqrt{m})$ . Exercise 5 from the previous homework will help you answer parts of this question.
  - (a) List the possible factorizations of  $(p)$  in  $\mathcal{O}_K$ . Justify your answer.
  - (b) Show that if  $p \mid \text{disc}(K)$ , then  $(p)$  ramifies in  $\mathcal{O}_K$ .
  - (c) Show that for an odd prime  $p$  not dividing  $m$ , we have
    - i.  $(p)$  is the product of two distinct ideals  $\iff m$  is a square modulo  $p$ .
    - ii.  $(p)$  is a prime ideal in  $\mathbb{Q}[\sqrt{m}] \iff m$  is not a square modulo  $p$ .
3. Let  $m \neq 1$  be a square-free integer which is 1 modulo 4. Let  $K := \mathbb{Q}(\sqrt{m})$ . Let  $p := 2$ . Show that
  - (a)  $(p)$  is the product of two distinct ideals  $\iff m \equiv 1 \pmod{8}$ .
  - (b)  $(p)$  is a prime ideal in  $\mathbb{Q}[\sqrt{m}] \iff m \equiv 5 \pmod{8}$ .
4. Marcus, page 91, Exercise 30, parts (a), (b), and (d).