Math 568 Homework 11
Spring 2009
Due: Thursday, April 9

1. Let $K := \mathbb{Q}[\sqrt{-1}, \sqrt{5}]$. Show that $\mathcal{O}_K = \mathbb{Z}[\sqrt{-1}, \frac{1+\sqrt{5}}{2}]$. Show that the only primes in $\mathbb{Z}$ that ramify in K are 2 and 5, and that their ramification indices are both 2. Deduce that K is unramified over $\mathbb{Q}[\sqrt{-5}]$. (Since we have already proved on a previous homework that $\mathbb{Q}[\sqrt{-5}]$ has class number 2, this shows that K is the *Hilbert class field of* $\mathbb{Q}[\sqrt{-5}]$.)

2. Show that $\mathbb{Q}[\sqrt{-47}]$ has class number 5.

3. Prove that the ideal class group of $\mathbb{Q}[\sqrt{-14}]$ is cyclic of order 4. (*Hint:* Show that there are either three or four ideal classes, and eliminate the cyclic group of order 3 and the Klein four group as possibilities for the class group.)

4. Prove that the ideal class group of $\mathbb{Q}[\sqrt{-21}]$ is the Klein four group.