Math 536 Homework 5
Spring 2008
Due: ***Wednesday***, February 20

1. Let p be a fixed prime.

   (a) Show that any ring with identity of order p^2 is commutative.

   (b) Show that there exist a noncommutative ring *without identity* of order p^2.

   (c) Show that there exist a noncommutative ring *with identity* of order p^3.

2. Let R be a simple ring, i.e., the only two sided ideals of R are R and (0). Let $\mathbb{M}_n(R)$ be the ring of $n \times n$ matrices over R. Show that $\mathbb{M}_n(R)$ is simple as well.

3. Let a, b be elements in a ring R. If $1 - ba$ is left-invertible, show that $1 - ab$ is left-invertible and construct a left-inverse for it explicitly. (Hint: $R(1 - ab)$ contains $Rb(1 - ab) = R(1 - ba)b = Rb$.)

4. Let R be a commutative ring with unit and suppose that N is an ideal of R such that $N^2 = 0$. Let $\phi : R \to R/N$ be the canonical map, and suppose that $e \in R/N$ satisfies $e^2 = e$. Show that there exists $d \in R$ with $\phi(d) = e$ and $d^2 = d$. (Hint: $(2e - 1)^2 = 1$.)