

Math 536 Homework 4
Spring 2008
Due: Friday, February 15

1. Let G be the abelian group defined by generators x, y , and z and relations

$$\begin{aligned}15x + 3y &= 0 \\3x + 7y + 4z &= 0 \\18x + 14y + 8z &= 0.\end{aligned}$$

Express G as a direct product of two cyclic groups.

2. Suppose that a finite group G has only one Sylow p -subgroup for each $p \mid |G|$. Show that G is a direct product of its Sylow p -subgroups.
3. Let G be a group of order 30. Show that G is not simple.
4. Find (up to isomorphism) all groups of order $2p$, and prove that your list is complete.
5. Let p, q be distinct primes. Prove that any group of order p^2q is solvable, and that one of its Sylow subgroups is normal.
6. Extra Credit: Let G be a simple group of order 60. Show that $G \cong A_5$.