Math 536 Homework 3
Spring 2008
Due: Friday, February 8

1. Let G be the group $\mathbb{Q}/\mathbb{Z}$.

   (a) Prove that every finitely generated subgroup of G is cyclic.

   (b) Show that for every positive integer t, G has a unique cyclic subgroup of order t.

2. Let A, B and C be finite abelian groups such that $A \times B$ and $A \times C$ are isomorphic. Prove that B and C are isomorphic.

3. Let A be a finitely generated abelian group, and let m be the maximum of the orders of the elements of A. Let S be the set of all elements of A whose order is m. Prove that A is generated by S.

4. Let G be a finite group and let N be a normal subgroup such that N and G/N have relatively prime orders.

   (a) Let H be a subgroup of G having the same order as G/N. Prove that $G = HN$.

   (b) Let g be an automorphism of G. Prove that $g(N) = N$.

5. Let G and H be finite groups of relatively prime order. Show that $\mathrm{Aut}(G \times H)$ is isomorphic to $\mathrm{Aut}(G) \times \mathrm{Aut}(H)$.