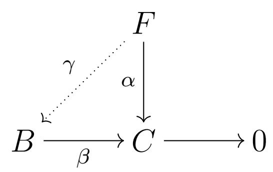
Math 536 Homework 2
Spring 2008
Due: Friday, February 1

1. Give an example of an integer n for which there are two groups of order n with composition series of different lengths.

2. Let G be a finite group and let p be the smallest prime dividing its order. Show that any subgroup of G of index p is normal.

3. Let $\beta : B \to C$ be a surjective homomorphism of groups. Show that if F is free abelian and $\alpha : F \to C$ is any homomorphism, then there exists a homomorphism $\gamma : F \to B$ making the diagram below commute, i.e. such that $\beta\gamma = \alpha$. (We say that F has the *projective property.*)



4. Use the previous exercise to deduce the following: If H is a subgroup of an abelian group G and G/H is free abelian, then H is a direct summand of G, that is there exists a subgroup K of G (with $K \cong G/H$) such that $G \cong H \times K$.

5. Suppose G is a finitely generated abelian group. Show that there exist finitely generated free abelian groups F_1, F_2 such that $G \cong F_1/F_2$.

6. Show that a group of order $2m$, with m odd, cannot be simple by showing that it contains a subgroup of index 2. (Hint: Use Cayley's theorem, Proposition 4.2, p.124 in Knapp.)