

Math 536 Homework 1
Spring 2008
Due: Friday, January 25

1. Let N be a normal subgroup of a group G . Suppose that there exists a subgroup K of G with $N \cap K = \{1\}$ and $G = N \cdot K$.

Any such K is called a *complement to N in G* .

- (a) Given G and N , is the complement K to N in G unique?
(b) Is K unique up to conjugation in G ?
(c) Is K unique up to isomorphism?
2. Construct all semidirect products of C_p by C_p for p prime. Here C_p denotes the cyclic group with p elements.
3. Let H_8 be the quaternion group of order 8, i.e. the group with presentation

$$\langle a, b : a^4 = 1, a^2 = b^2, b^{-1}ab = a^{-1} \rangle .$$

Show that H_8 is **not** a semidirect product of a group of order 4 by a group of order 2.

4. For each odd prime p construct a nonabelian group G of order p^3 and exponent p . (Hint: Construct G as a semidirect product of K by Q , where K is a certain group of order p^2 and Q is cyclic of order p .)
5. Let T be a group of order 12 which is generated by two elements s and t such that $s^6 = 1$ and $t^2 = s^3 = (st)^2$. Prove that such a group T exists by constructing it as a semidirect product of \mathbb{Z}_3 by \mathbb{Z}_4 .
6. Find a composition series for the following groups:
- (a) the quaternion group H_8 ;
(b) the dihedral group D_6 ; and
(c) $A_4 \times A_4$.