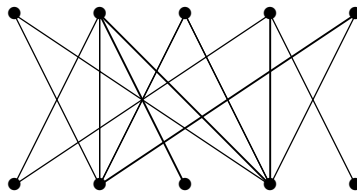
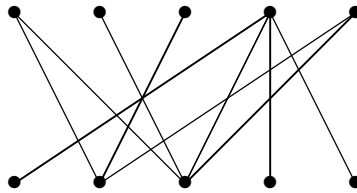


Math 485 Homework 9
Fall 2007
Due: Friday, November 16

In all the problems, indicate how you arrived at your answer and explain your work.

- For each graph below, find a maximum matching and a minimum vertex cover and explain why they are optimal.



- For each of the graphs in problem (1), find a maximum independent set and a minimum edge cover.
- Which simple graphs have the following properties?
 - $\alpha(G) = 1$ (where $\alpha(G)$ is the maximum size of an independent set)
 - $\alpha'(G) = 1$ (where $\alpha'(G)$ is the maximum size of a matching)
 - $\beta(G) = 1$ (where $\beta(G)$ is the minimum size of a vertex cover)
 - $\beta'(G) = 1$ (where $\beta'(G)$ is the minimum size of an edge cover)
- Show that if M is a matching in a graph G with $\#M < \alpha'(G)/2$, then M is not maximal.
- Show that any bipartite graph G has a matching M with $\#M \geq e(G)/\Delta(G)$. (Here $e(G)$ is the number of edges in G and $\Delta(G)$ is the maximum degree among all vertices of G .)