

**Math 485 Homework 6**  
**Fall 2007**  
**Due: Friday, October 19**

In all the problems, indicate how you arrived at your answer and explain your work.

1. We proved in class that the center of a tree is a single vertex or a single edge.
  - (a) (5 points) Find an example of a connected graph whose center is disconnected, i.e. there are two vertices  $u$  and  $v$  in the center such that no  $u, v$ -path is contained in the center.
  - (b) (10 points) Suppose  $G$  is a connected graph and  $u$  and  $v$  are vertices in distinct connected components of the center of  $G$ . Show that  $G$  contains a cycle which includes both  $u$  and  $v$ .
  
2. Describe what sorts of trees have Prüfer codes of the following forms:
  - (a) (5 points) All entries of the code are the same.
  - (b) (5 points) All entries of the code are distinct.
  
3. Use the previous result to determine how many trees with vertex set  $\{v_1, \dots, v_n\}$  have the following properties:
  - (a) (5 points) Trees with exactly 2 leaves.
  - (b) (5 points) Trees with  $n - 1$  leaves.
  
4. (15 points) For  $n \geq 3$ , let  $G_n$  be the graph obtained by deleting a single edge from  $K_n$ . Use Cayley's formula to show that the number of spanning trees of  $G_n$  is  $(n - 2) \cdot n^{(n-3)}$ .
  
5. (10 points) Let  $T$  be a tree. Prove that the vertices of  $T$  all have odd degree if and only if for all  $e \in E(T)$ , both components of  $T - e$  have an odd number of vertices.