

Math 485 Homework 4
Fall 2007
Due: Friday, September 28

In all the problems, indicate how you arrived at your answer and explain your work.

1. (a) How many subgraphs does K_n have that are isomorphic to the n -cycle C_n ?
(b) How many subgraphs does $K_{n,n}$ have that are isomorphic to C_{2n} ?
2. Find the maximum number of edges in a disconnected n -vertex graph.
3. (a) Prove that the maximum number of edges in a bipartite subgraph of the Petersen graph is ≤ 13 .
(b) Find a bipartite subgraph of the Petersen graph with 12 edges. (This is actually the maximum number of edges such a subgraph can have, see the extra credit problem below.)
4. Show that a loopless graph G has a 3-partite subgraph H such that

$$e(H) \geq \frac{2}{3}e(G).$$

Here, for a graph G , $e(G)$ denotes the number of edges in G .

5. Prove that every cycle of length $2r$ in a hypercube Q_n is contained in a subcube of dimension at most r .
6. Determine which of the following sequences are graphic. For each one that is graphic, find a graph whose degree sequence is the given sequence.
 - (a) (3, 3, 2, 2, 1, 1)
 - (b) (6, 5, 5, 2, 2, 2, 2)
 - (c) (5, 5, 5, 4, 2, 1, 1, 1)
 - (d) (5, 5, 4, 4, 2, 2, 1, 1)
7. **Extra Credit:** Prove that the Petersen graph has no bipartite subgraph with 13 edges.