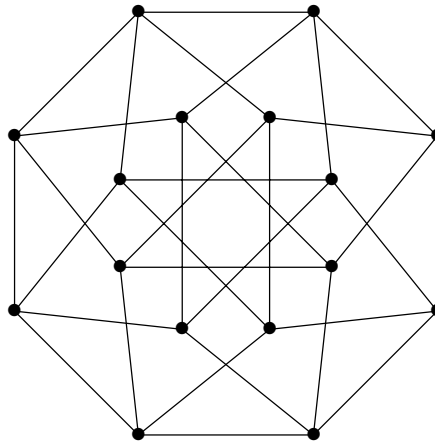


Math 485 Homework 3
Fall 2007
Due: Friday, September 21

In all the problems, indicate how you arrived at your answer and explain your work.

1. For each $n \geq 1$ define the following graph H_n : The vertices of H_n are n -tuples whose entries are either 0 or 1. Two vertices are adjacent if and only if they differ on exactly **two** coordinates.
 - (a) Draw a picture of H_2 and of H_3 .
 - (b) How many components does H_n have for each $n \geq 1$?
2. For which values of n and m is the complete bipartite graph $K_{n,m}$ Eulerian?
3. Consider the following graph G :



- (a) Give a decomposition of G into cycles.
 - (b) Find an Eulerian circuit in G .
4. Let P and Q be paths of maximum length in a connected graph G . Prove that P and Q have a common vertex.
 5. Prove that every n -vertex graph with at least n edges contains a cycle.