

Math 485 Homework 2
Fall 2007
Due: Friday, September 14

In all the problems, indicate how you arrived at your answer and explain your work.

1. Let Q_n be the n -cube which is defined as follows: The vertices of Q_n are n -tuples whose entries are either 0 or 1. Two vertices are adjacent if and only if they differ on exactly one coordinate. Show that Q_n is bipartite for each $n \geq 1$.
2. Let G be a loop-less graph such that each vertex has degree at least 3. Show that G contains a cycle of even length.
3. Let G be a simple graph, and v a cut vertex of G . Show that $\overline{G} - v$ is connected.
4. Prove that a simple graph G with n vertices and k components can have at most $(n - k) \cdot (n - k + 1)/2$ edges. Deduce from this that G must be connected if it has more than $(n - 1) \cdot (n - 2)/2$ edges.
5. Prove that the number of vertices of a self-complementary graph must be of the form $4k$ or $4k + 1$ where k is an integer.
6. Let W be a closed walk of length at least 1 which does not contain a cycle. Show that there is some edge e which repeats immediately in W , i.e. the walk follows e in one direction and then immediately follows it back in the other direction.