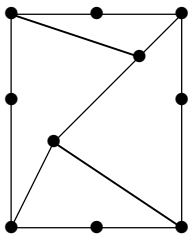
# Math 485 Homework 10
# Fall 2007
# Due: Friday, November 30

In all the problems, indicate how you arrived at your answer and explain your work.

1. Prove or disprove: Every vertex cover of a graph contains a minimum vertex cover.
2. Prove or disprove: Every tree has at most one perfect matching.
3. Show that $\alpha(G) \geq \frac{n(G)}{\Delta(G)+1}$ for any graph $G$.
4. (a) Exercise 3.3.1 on page 145 of the textbook.
   (b) Determine whether the graph below has a 1-factor.



5. (a) For every graph $G$ prove that $\beta(G) \leq 2 \cdot \alpha'(G)$.
   (b) For every $k \geq 1$ construct a simple graph $G$ with $\alpha'(g) = k$ and $\beta(G) = 2k$.
6. Let $T$ be a tree. Show that $T$ has a perfect matching if and only if $o(T - v) = 1$ for every vertex $v \in V(T)$.