Writing Mathematically

1 Introduction

In addition to becoming familiar with the art of counting, one of the goals of this class is to improve your mathematical writing/communication skills. To that end, I have written up a few things to keep in mind while writing homework solutions.

One of the most important things to remember is that writing a mathematical document is no different from writing any other type of document. **In particular, always use complete sentences when writing up your solutions.** Treat mathematical statements as if they were written in English. When read, sentences involving mathematical formulas and symbols should be grammatically correct and complete. After you have written up your solution, **read it out loud.** Many times you will be able to pick up on awkwardly worded phrases and/or mathematical errors that you would not have seen otherwise.

Do not use symbols like $=$, $\exists$ or $\forall$ as a shortcut for written English. A general rule of thumb is that a statement should be written entirely using mathematical symbols or written entirely in English. Try not to write a statement that mixes the two. For example, instead of writing "let n be an integer ≥ 3", write "let n be an integer greater than or equal to 3" or "let n be an integer such that $n \geq 3$". In the last example, "$n \geq 3$" is a complete statement written entirely with mathematical symbols as opposed to "≥ 3", which is not a complete statement.

Another similarity between mathematical writing and any other type of writing is that rarely is a well organized document written in a single draft. You may go through several drafts where you simply write down any idea and/or calculation that comes to mind. Ultimately, much of this work may not show up in your final polished paper, however without it, the final result would not be as well thought out. **Remember, your final solution should not be a re–telling of how you discovered the solution, but rather a concise explanation of what the solution is.**

If you are having trouble finding a solution, start by writing down phrases/calculations/definitions that you think may be relevant to the problem. Try to see how any of these ideas can be combined to give you a new piece of information that may (or may not) lead you closer to a solution. I highly recommend finding an empty classroom and working out the problem on the chalkboard. Not only does this save paper, but you are more likely to write things down on the board that you wouldn't have written down on a sheet of paper. Often times the only thing separating a student from a solution is the simple act of writing something down so that you can physically see it.

Once you have found a solution, read through all of your work. Make sure that you have answered the question, explained every detail and eliminated any unnecessary steps. Many of your initial ideas/calculations may no longer be relevant. Do not include them in your final solution. Make sure to organize each step in a logical order so that the reader can easily follow. Then and only then should you start writing up your final solution to be handed in.

2 When Writing Your Final Solution...

1. **Clearly state what you are going to prove and what method of proof you will use.** It makes it much easier for the reader to follow along if they have some sense of where you are headed and how you intend to get there. Note that this does not necessarily mean you have to rewrite the statement of the problem.

2. **Know who your audience is.** Always keep in mind who will be reading your paper/solution. This will give you an idea of the level of detail that is required. For this class, you may assume that your audience is your fellow classmates who are familiar with the definitions and theorems from class. **In other words, you do not need to restate definitions and/or theorems.**

3. **Say precisely what you mean and mean precisely what you say.** Be very careful in how you phrase each sentence. **Make sure that you are using proper notation and terminology at all times.** There is a tendency to paraphrase definitions and theorems. In the beginning, try to resist that urge until you completely understand them. Definitions and theorems are worded very precisely and hence changing one word could easily change its meaning and/or validity.

4. **Define your notation.** If you introduce any new notation, you must define it. Even if you think it's meaning is clear from the context, you cannot assume that it will be clear to the reader. Do not define new notation that conflicts with notation already established in class or in the text.

5. **Use examples appropriately.** During your proof, you may decide to introduce some new notation, make a definition, or describe an algorithm. If you are finding it difficult to describe precisely what you mean, an example can be a valuable tool to get your point across. **However, never use an example as a substitute for a formal definition or proof.**

6. **Be specific.** Many times you may be tempted to write a phrase like "there are 4 choices." Choices of what? Be more specific. Try instead something like "There are 4 ways to select the suit of a straight flush."

7. **Always state your conclusion at the end of the proof.** If you are asked to compute the number of ways to place n distinct objects into 2 identical boxes, then the final line of your proof should be something like "Therefore, there are 2^{n-1} ways to place n distinct objects into 2 identical boxes."

8. **Use phrases such as:**

   (a) *Consequently, therefore, thus, yields,* or *produces* to indicate an important conclusion is coming next.

   (b) *As claimed* or *as desired* to inform the reader that we have done what we set out to do.

   (c) *Let n be an arbitrary positive integer* to define notation.

   (d) *We will proceed by...* induction, contradiction, etc.

   (e) *It remains to show that...* when you have one last thing to prove.

9. **Avoid phrases like:**

   (a) *Obviously...* Many times this phrase is used to avoid explaining something that the author feels the reader should already know. If it really is obvious, then it should be easy to explain. Explain it!

   (b) *using the same logic...* If you really are using the same logic, then you should be able to reorganize your work so that you do not need this separate case that uses the same logic. If you are using *similar* logic, make sure to explain what the differences are and how it changes your *logic.*

   (c) *if you think about it,...* This is not the job of the reader. It's your job to explain *every* detail.

   (d) *and so on and so forth...* Make sure to be as specific as possible, especially when describing some sort of pattern. Do not leave it to the reader to guess what the pattern is. Explain the general case of the pattern.

   (e) *this can be written as...* Be more descriptive. Instead of writing "this can be written as $52 \cdot 51/2$ ways", try writing "thus, there are $52 \cdot 51/2$ ways to select two cards from a standard deck of 52."