

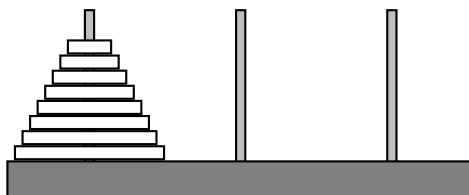
Recurrence Relations

For each of the following sequences, compute the value of the first four terms ($n = 0, 1, 2, 3$) by hand. Start to think about how you can use the values you've computed to determine the value of the next couple of terms ($n = 4, 5$) in the sequence. Try to find a recurrence relation that can be used to find the value of any term in the sequence.

1. Let b_n represent the number of binary sequences of length n that do not contain consecutive ones.

Hint: How many sequences of length n end in a zero? How many sequences of length n end in a one?

2. The n disk Tower of Hanoi puzzle consists of three pegs and n disks of varying diameter. The puzzle starts with all n disks placed on one of the three pegs (usually the left-most peg). The disks are arranged in increasing order from top to bottom according to their diameters. That is, the largest disk is on the bottom and the smallest disk is on the top, as illustrated below in the case $n = 8$.



The goal of the puzzle is to move all n disks to one of the other two pegs (usually the right-most peg) by moving only one disk at a time and never placing a larger disk on top of a smaller disk.

Let h_n represent the *minimum* number of moves required to solve the n disk Tower of Hanoi puzzle.

Hint: Think about what state the puzzle must be in so that you can move the largest disk.

3. Let r_n be the *maximum* number of regions the xy -plane can be divided into using n straight lines.

Hint: Start by drawing a single line. Any line will do. Now draw a second line. What property must this line have in order to create the maximum number of regions? Draw a third line. What property must this line have? Keep adding more lines to your picture. Each time you draw a line, draw it slowly and notice exactly when a new region of the xy -plane is created.

4. Suppose that you have n letters addressed to n distinct people. Let d_n be the number of ways to deliver the letters to the n people so that everyone receives exactly one letter, but nobody receives the letter that is addressed to them.

Hint: Suppose that you are one of the n people. In how many ways can the letters be delivered so that you receive the letter addressed to the person that received the letter addressed to you? In how many ways can the letters be delivered so that you do not receive the letter addressed to the person that received the letter addressed to you?