Denker **Spring 2009**

519 Stochastic Processes- Assignment 3

Due Date: Tuesday, 20–10–2009

Problem 1: (Exercise 2.1.4) Show that the following processes, whose values at each $t \geq 0$ are given below, are all martingales:

1. $C(t) = \sigma B(t)$, where $B(t)$ is standard Brownian motion in $\mathbb{R}^m$ and σ is a $d \times m$ matrix.
2. $\|C(t)|^2 - tr(A)t$ where $A = \sigma\sigma^T$.
3. $\exp[(u, C(t)) - \frac{1}{2}(u, Au)t]$ where $u \in \mathbb{R}^d$.
4. $\widetilde{N}(t)^2$ where $\widetilde{N}$ is a compensated Poisson process with intensity λ.
5. $\widetilde{N}(t)^2 - \lambda t$.

Problem 2: (Exercise 2.2.2) If S and T are stopping times and $\alpha \geq 1$, show that $S + T$, αT, $\min(S, T)$ and $\max(S, T)$ are also stopping times.

Problem 3: (Exercises 2.2.5 and 2.2.6) Let M, M_1, M_2 and N be martingales in $\mathcal{M}$. Show that

1. $M(t)N(t) - < M, N > (t)$ is a martingale.
2. $< \alpha M_1 + \beta M_2, N >= \alpha < M_1, N > +\beta < M_2, N >$ for $\alpha, \beta \in \mathbb{R}$.
3. $E(< M, N > (t))^2 \leq E(< M, M >)E(< N, N >)$ with equality iff $M(t) = cN(t)$ a.s. for some $c \in \mathbb{R}$.

Problem 4: (Exercise 2.3.3) Show that $\sum_{0 \leq s \leq t} |\Delta X(s)| < \infty$ a.s. if X is a compound Poisson process.

Problem 5: (Exercise 2.3.17) Show that every subordinator is of finite variation.