August 4, 2006 Math 035 Quiz #3 Name: _____

The number in brackets gives how many points the question is worth. You do not need to show your work unless asked to do so, but as always, it might help you get part marks. You have 30 minutes for this quiz; calculators are permitted, but no books, notes, or foul-mouthed stand-up comics are allowed.

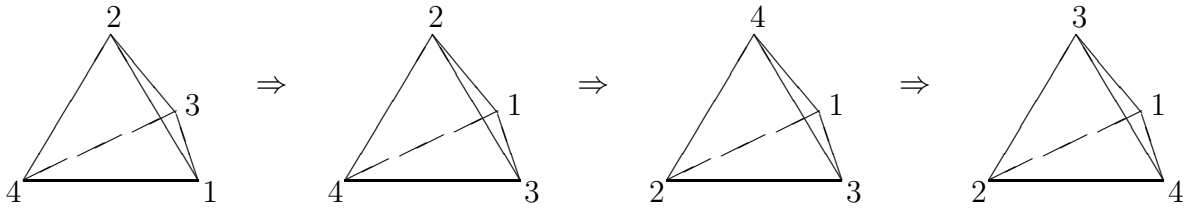
(6) 1. Two-parter, comin' at ya:

(a) For each of the two tetrahedra shown, determine if it is right-handed or left-handed.

Solution: The first tetrahedron is left-handed; the second is right-handed.

(b) How many switches does it take to get from one to the other? Draw the new labeling after each switch, and for each of the intermediate tetrahedra, determine whether it is right-handed or left-handed. *Draw a new tetrahedron for each step along the way.*

Solution: It takes three switches - one possibility (of many) is



Of the four tetrahedra shown, the first and third are left-handed; the second and fourth are right-handed.

(7) 2. And now, an even-more-parter...

(a) For each of the two configurations of the 15-puzzle drawn below, write out the list of numbers used to determine B, and calculate B, the number of backwards pairs.

I.

3	7	15	1
2	9	4	
6	5	11	10
12	8	14	13

II.

1	2	3	5
8	13	6	4
10	14	9	
12	7	11	15

Solution: For the first configuration, $B = 37$, as shown below:

3	7	15	1	2	9	4	16	6	5	11	10	12	8	14	13
2	5	12	0	0	4	0	8	1	0	2	1	1	0	1	0
		7	19			23		31	32		34	35	36		37

Remember that each number in the second row represents how many numbers after that point in the list are smaller than the number we are investigating, and the third row is just to help us add them all up. For the second configuration, we get $B = 25$:

1	2	3	5	8	13	6	4	10	14	9	16	12	7	11	15
0	0	0	1	3	7	1	0	2	4	1	4	2	0	0	0
				4	11	12		14	18	19	23	25			

(b) Would it take an even or an odd number of switches to go between these two lists?

Solution: Since the number of backwards pairs in each list is odd, and each switch reverses the parity of that number, it would take an even number of switches to go between them.

(c) Say we colour the 16 squares in a chess-board fashion, and the top left square is shaded. Then what colour is the empty square in each of the above configurations?

Solution: In the first configuration, the empty square will be shaded; in the second, it will be unshaded.

(d) Calculate A for each of the two configurations given.

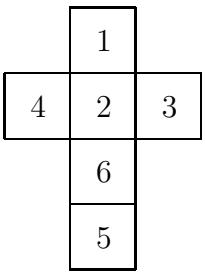
Solution: $A = 38$ for the first configuration, and $A = 25$ for the second, because of part (c).

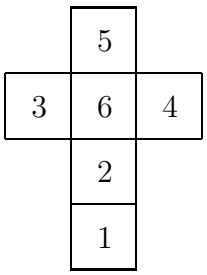
(e) Is it possible to get from one to the other using only legal moves?

Solution: A legal move does not change the parity of A, so it is not possible to go between these two configurations, because the first has an even value of A, and the second has an odd value.

(5) 3. Just two parts to this one. And it's about dice!

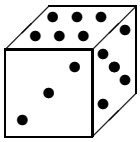
(a) For each of the two (unfolded) dice shown, determine whether it is right-handed or left-handed.

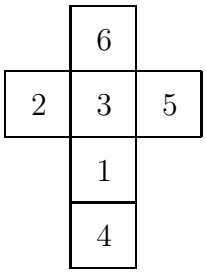




Solution: By folding up the first one and looking at it from the upper right, we can see that the numbers 1, 2, 3 increase in the counterclockwise direction, so the die is right-handed. For the second die, we can either look at it from the lower left rear and see that it is also right-handed, or we can notice that switching 1 and 6 reverses the orientation and gives us a left-handed die, so the original die must have been right-handed.

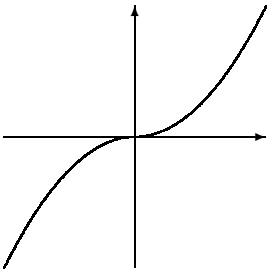
(b) Consider the die pictured below. Using the fact that opposite sides must add up to 7, determine the missing sides and fill in the five blank faces in the unfolded diagram at right, then determine the orientation of this die.

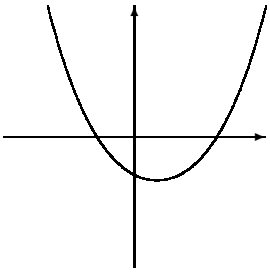




Solution: Looking from the lower left indicates that the die is left-handed; it is completed as shown.

(2) 4. For each graph shown, indicate whether the polynomial it represents has even or odd degree.





Solution: The first polynomial has odd degree (since the ends go in opposite directions), while the second has even degree (since the ends go in the same direction).