July 7, 2006 Math 035 Quiz #1 Name: ______________________

The number in brackets gives how many points the question is worth. Show your work on all questions. If no work is shown, you will not receive full credit for your answer, even if it is right; if you convince me you understand some of what's going on, you will receive part marks, even if your answer is wrong. You have 30 minutes for this quiz; calculators are permitted, but no books, notes, or French horns are allowed.

(5) 1. I quit my job and take up sumo wrestling. Since I do not weigh several hundred pounds, there is a $\frac{4}{5}$ chance that I lose any given bout I find myself in. If I enter a best-of-three series with Ozeki Kaio, in which the series ends as soon as one of us has won two bouts, what is the probability that the series lasts all three bouts?

Solution: The only two ways for the series not to last all three bouts is for one of us to win both of the first two. Using the multiplication rule, we have

$$\begin{aligned} \text{P(he wins first two)} &= \text{P(he wins the first and the second)} \\ &= \text{P(he wins the first)} \cdot \text{P(he wins the second)} \\ &= \tfrac{4}{5} \cdot \tfrac{4}{5} = \tfrac{16}{25} \\ \text{P(I win first two)} &= \text{P(I win the first and the second)} \\ &= \text{P(I win the first)} \cdot \text{P(I win the second)} \\ &= \tfrac{1}{5} \cdot \tfrac{1}{5} = \tfrac{1}{25} \end{aligned}$$

Now we can use the addition rule to find out that

$$\begin{aligned} \text{P(the series ends in two bouts)} &= \text{P(one of us wins the first two)} \\ &= \text{P(he wins first two)} + \text{P(I win first two)} \\ &= \tfrac{16}{25} + \tfrac{1}{25} \\ &= \tfrac{17}{25} \end{aligned}$$

Finally, the subtraction rule tells us that

$$\begin{aligned} \text{P(the series lasts three bouts)} &= 1- \text{P(it ends in two)} \\ &= 1 \text{ - } \tfrac{17}{25} \\ &= \tfrac{25}{25} - \tfrac{17}{25} \\ &= \tfrac{8}{25} \end{aligned}$$

Alternately, we could make a tree showing all of the possible outcomes, just as in the example from class where we considered the best-of-three series between two evenly matched baseball teams.

(5) 2. Not to be discouraged by my failure in the sumo ring, I turn to football; my team gets the ball on our own 40-yard line with one drive to go, needing a touchdown to win the game. For simplicity's sake, assume that on each first down sequence we either get exactly ten yards for another first down, or lose the ball. Thus, to get a touchdown and win the game we need six first downs in a row; if we don't get them, we lose the game.

(a) If on any given first down sequence, the probability that the defense stops us, so that we do *not* get another first down, is 17%, what is the probability that we win, by getting six consecutive first downs?

(b) What is the probability that we lose, by failing to get six consecutive first downs?

Solution: Since the probability of not getting a first down on any given sequence is $17\% = 0.17$, the probability that we *do* get one is $0.83 = 83\%$. By the multiplication rule, the probability of doing so on six consecutive sequences is $(0.83)^6 = 0.327 = 32.7\%$. This is the probability that we win, so the probability that we lose is $1 - 0.327 = .673 = 67.3\%$.

(6) 3. Eventually I turn to basketball. Since I'm nowhere near tall enough to do anything else, like dunk, I practice shooting for hours on end, and become a 90% free throw shooter - that is, I have a $\frac{9}{10}$ chance of making any given free throw. As I launch a buzzer-beating fade-away two-point shot over some defender guy (probably Shaq - we'll say Shaq), with my team losing by a point, he fouls me, and I get two free throws. Of these two shots, what is the probability that I

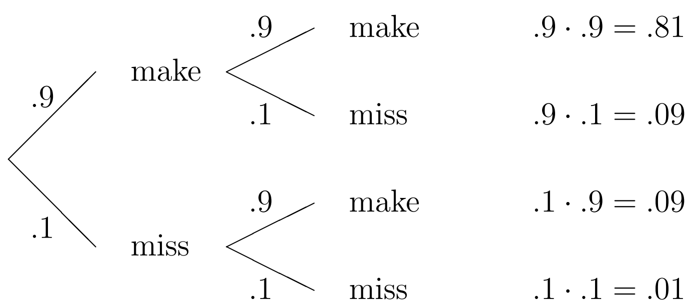
(a) miss both (causing us to lose the game)? ________

(b) make one and miss one (sending us to overtime in a tie)? ________

(c) make both (launching us to victory)? ________

Hint: Draw a tree showing all the possible sequences of makes and misses.

Solutions: As suggested, draw a tree enumerating all the possibilities:



Thus there is a $.81 = 81\%$ chance that I make both free throws, a $.01 = 1\%$ chance that I miss both, and a $.09 + .09 = .18 = 18\%$ chance that I make one and miss one. (We must add together the two different ways in which this can happen to obtain the total probability of the event "I make one and miss one.")

(2) 4. In the end, though, my experiences with sports drive me to gamble, so I get in the car and drive to Las Vegas. What is the probability that I lose my entire life savings at the poker table if

(a) The odds *in favour* of this happening are 9:4? ________

(b) The odds *against* this are 9:4? ________

Solution: Recall that if the odds in favour of event A are $a : b$, then $P(A) = \frac{a}{a+b}$. Conversely, if the odds *against* A are $a : b$, then $P(A) = \frac{b}{a+b}$. So in the first case, the probability of my total financial ruin is $\frac{9}{9+4} = \frac{9}{13}$; in the second case, it is $\frac{4}{9+4} = \frac{4}{13}$.

(2) 5. If I have a 10% chance of getting a speeding ticket on the drive back from Vegas, what are the odds *against* me getting a ticket?

Solution: Recall that the odds against event A happening are given by

$$\frac{P(\text{not } A)}{P(A)}$$

so the odds against me getting a ticket are $\frac{90\%}{10\%} = \frac{9}{1}$, or 9 to 1.