August 2, 2006 Math 035 Assignment #7 Solutions

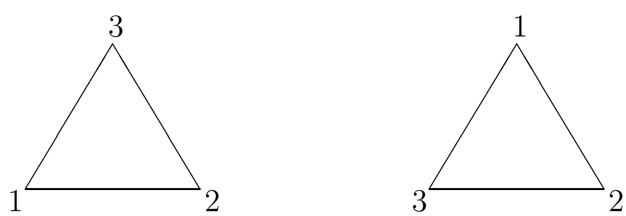
Each question is worth 5 points, for a total of 20.

1. In each case, use the rules governing how even and odd numbers multiply to determine whether the number given is even or odd. k stands for any natural number.
(a) $2 \cdot (k + 8) + 3$
(b) $7 \cdot (2k + 1) - 2k$
(c) $-2 \cdot (k - 1) + 18$

Solution:
(a) An even number times anything else equals an even number, so $2 \cdot (k + 8)$ is even. Even + odd = odd, so the final answer is odd.
(b) $2k$ is even, so $2k + 1$ is odd. Odd $\times$ odd = odd, so $7 \cdot (2k + 1)$ is odd. Odd - even = odd, so the final answer is odd.
(c) -2 is even, so $-2 \cdot (k - 1)$ is even. Even + even = even, so the final answer is even.

2. Use the distinction between clockwise and counterclockwise orientation to determine which pairs of triangle labellings below are equivalent. Remember that two labelled triangles are equivalent if we can go from one to the other by moving it around in the plane, without picking it up and flipping it over.

(a)



(b)

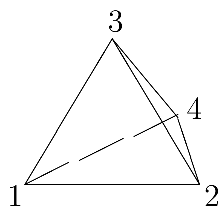
(c)

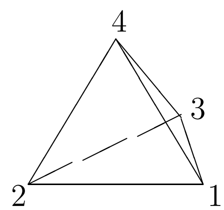
Solution:
(a) The first triangle has counterclockwise orientation; the second is oriented clockwise. So the two labellings are *not* equivalent.
(b) Both triangles are oriented counterclockwise, so the labellings are equivalent.
(c) Relabel blue as 1, red as 2, and yellow as 3; then both triangles are oriented counterclockwise, and the labellings are equivalent. If we had chosen a different relabelling (for

example, blue as 1, yellow as 2, red as 3), we might have had both oriented clockwise, but they would still be equivalent.

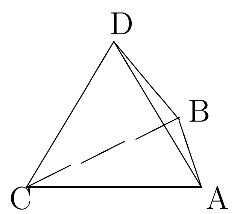
3. Use the distinction between right-handed and left-handed orientations to tell which pairs of labellings of a tetrahedron are equivalent. Recall that two labelled tetrahedra are equivalent if we can go from one to the other by movements and rotations in space.

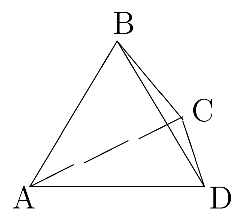
(a)



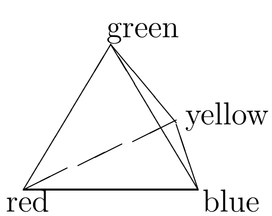


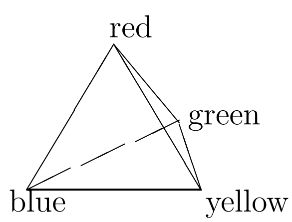
(b)





(c)

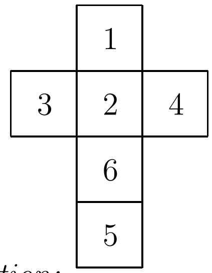




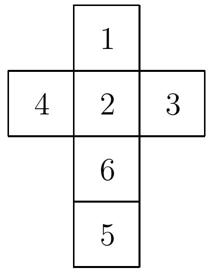
Solution:
(a) Both labellings are right-handed, so they are equivalent.
(b) The first labelling is left-handed; the second is right-handed. They are *not* equivalent.
(c) Relabel green as 1, yellow as 2, red as 3, and blue as 4; then the first is right-handed and the second is left-handed, so they are *not* equivalent. Again, by a different choice of relabelling we might have gotten the first as left-handed and the second as right-handed, but they would still not be equivalent.

4. We can also define handedness for dice. Say a die is *right-handed* if, when you look at the corner between the numbers 1, 2, and 3, those three numbers increase counterclockwise, and *left-handed* if going from 1 to 2 to 3 takes you in a clockwise direction. Each diagram below represents a die - if you have trouble visualising it, copy the diagram, cut it out, and fold it into a cube. For each die shown, determine whether it is right-handed or left-handed.

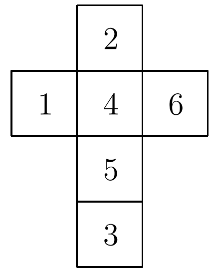
(a)



(b)



(c)



Solution:

(a) Left-handed

(b) Right-handed

(c) Right-handed