July 18, 2006 Math 035 Assignment #4 Solutions

p. 26-28, #2,3,5,6,21

2. Which of the numbers 57, 63, 67, 69, 87, 91, 97, 101, 109, and 111 are prime? Which are composite?

Solution: 67, 97, 101, and 109 are prime; the others are all composite, and have the following factorisations:

$$\begin{aligned} 57 &= 3 \cdot 19 \\ 63 &= 3 \cdot 21 \\ &= 7 \cdot 9 \\ 69 &= 3 \cdot 23 \\ 87 &= 3 \cdot 29 \\ 91 &= 7 \cdot 13 \\ 111 &= 3 \cdot 37 \end{aligned}$$

3. Some primes can be written in the form $1 + N^2$ for some natural number N. For example, $5 = 1 + 2^2$, and $17 = 1 + 4^2$.
(a) Find three more primes of the form $1 + N^2$.
(b) Do you think that there is an end to such primes?

Solution: $N = 1, 6, 10, 14, 16$ gives

$$\begin{aligned} 2 &= 1^2 + 1 \\ 37 &= 6^2 + 1 \\ 101 &= 10^2 + 1 \\ 197 &= 14^2 + 1 \\ 257 &= 16^2 + 1 \end{aligned}$$

and each of these is prime (though only three of them are required for a complete answer to the question).

No one knows whether or not there is an end to such primes.

5. Some primes are 1 more than a power of 2; for example, $5 = 1 + 2^2$, and $17 = 1 + 2^4$.
(a) Find another such prime.
(b) Do you think that there is an end to such primes?

Solution: The next such prime is $257 = 2^8 + 1$, since

$$\begin{aligned} 2^5 + 1 &= 33 = 3 \cdot 11 \\ 2^6 + 1 &= 65 = 5 \cdot 13 \\ 2^7 + 1 &= 129 = 3 \cdot 43 \end{aligned}$$

$2 = 2^0 + 1$ would also be a correct answer, although it somewhat misses the point...

Again, no one knows whether or not there is an end to these.

6. Some primes are 1 less than a power of 2; for example, $3 = 2^2 - 1$, and $7 = 2^3 - 1$.
(a) Find two more such primes. (b) Do you think that there is an end to such primes?

Solution: $31 = 2^5 - 1$, and $127 = 2^7 - 1$ are the next two; $511 = 2^9 - 1$ is also a prime of this form. Just as with the other two kinds, no one knows the anwer to part (b).

21. Write 57 and 117 as the product of primes.

Solution:

$$\begin{aligned} 57 &= 3 \cdot 19 \\ 117 &= 3 \cdot 39 \\ &= 3 \cdot 3 \cdot 13 \\ &= 3^2 \cdot 13 \end{aligned}$$