July 11, 2006 Math 035 Assignment #3 Solutions

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23. (a) What is the probability of scoring a 4 before a 7 with two dice?

Solution: Let A be the event of rolling a 4 or a 7, and B the event of rolling a 4. The question asks for $P(B|A)$, that is, the probability of rolling a 4 given that we have just rolled a 4 or a 7. Of the 36 ways that 2 dice can land, there are 3 that yield a total of 4, and 6 that yield a total of 7. Thus there are 9 outcomes which give event A; of these 9, only 3 give event B, so the desired probability is $\frac{3}{9} = \frac{1}{3}$.

(b) The *usual* house payoff is 9 to 5. That is, the house pays 9 dollars for a 5-dollar wager, if the player "shoots the 4." What would *fair* odds be?

Solution: The probability of winning on any given roll is $\frac{1}{3}$, so the odds against winning are 2 to 1, meaning that the house should pay out twice as much as the amount the player bets; for a 5-dollar wager, the house should pay 10 dollars for the odds to be fair.

24. If you wager 1 dollar that you will shoot a 4 before a 7 (see above), what is your expectation? The house payoff is 9 to 5.

Solution: Your probability of winning is $\frac{1}{3}$, and if you win, you are paid $\frac{9}{5} =$ 1.8 dollars for your 1-dollar bet. Thus your expected gain is $\frac{1}{3} \cdot \$1.80 = \$0.60$.

Your probability of losing is $\frac{2}{3}$, and if you lose, you lose your 1-dollar bet. So your expected loss is $\frac{2}{3} \cdot \$1 = \frac{2}{3}$ dollar.

Taking the difference of these gives your expectation:

$$\begin{aligned} \text{expectation} &= \text{expected gain - expected loss} \\ &= \frac{3}{5} - \frac{2}{3} \text{ dollars} \\ &= \frac{9}{15} - \frac{10}{15} \text{ dollars} \\ &= -\frac{1}{15} \text{ dollars} \end{aligned}$$

25. (a) What is the probability of scoring a 6 before a 7 with two dice?

Solution: There are 5 ways of scoring a 6, and 6 ways of scoring a 7, so exactly as above, the probability of the 6 occurring first is $\frac{5}{5+6} = \frac{5}{11}$.

(b) If the house pays 7 dollars for 6 dollars if the shooter wins, what is the expectation of the shooter when he wagers 1 dollar?

Solution: His expected gain is $\frac{5}{11} \cdot \frac{7}{6} = \frac{35}{66}$ dollars; his expected loss is $\frac{6}{11} \cdot 1 = \frac{6}{11}$ dollars. Thus his expectation is $\frac{35}{66} - \frac{6}{11} = \frac{35}{66} - \frac{36}{66} = -\frac{1}{66}$ dollars.

(c) This gamble is a better deal for the shooter than the gamble of exercise 24, since he does not lose as much money, on average, per bet.

26. A bet of "big eight" is the wager that in two consecutive rolls of two dice neither a 7 nor an 8 appears.
(a) What is the probability of winning the "big eight" bet?

Solution: There are 6 ways of rolling a 7, and 5 ways of rolling an 8, so the probability that one or the other rolls is $\frac{11}{36}$. Then the subtraction rule tells us that the probability neither appears on any given roll is $1 - \frac{11}{36} = \frac{25}{36}$. Thus by the multiplication rule,

$$\begin{aligned}
\text{P(winning the bet)} &= \text{P(neither 7 nor 8 appears on either roll)} \\
&= \text{P(neither appears on first roll)} \\
&\quad \cdot \text{P(neither appears on second roll)} \\
&= \tfrac{25}{36} \cdot \tfrac{25}{36} \\
&= \tfrac{625}{1296} = .482
\end{aligned}$$

(b) Is a payment of 1 to 1 fair? If not, is a 1-to-1 payment to the advantage of the house or the player?

Solution: The probability of the player winning the bet is less than 50%, so the payment is not fair, and favours the house.

(c) What is a fair payoff?

Solution: The probability that the player loses the bet is $1 - \frac{625}{1296} = \frac{671}{1296}$, so the odds against him are actually 671 to 625. Thus a fair payoff would be \$671 for every \$625 bet, or $\frac{671}{625} = 1.07$ dollars for every dollar bet.

(d) What is the expected gain (or loss) per one-chip bet?

Note: This question should simply ask for the expectation; in class we used the terms "expected gain" and "expected loss" to mean something slightly different, namely the two quantities which are calculated to find the expectation. Answers should be graded accordingly.

Solution: The expected gain is $\frac{625}{1296} \cdot 1 = \frac{625}{1296}$; the expected loss is $\frac{671}{1296} \cdot 1 = \frac{625}{1296}$. Thus the expectation is $\frac{625}{1296} - \frac{671}{1296} = -\frac{46}{1296} = -\frac{23}{648}$ of a chip, or -.035 chips.

33. The records of a publisher show that 25 percent of her books break even, 25 percent lose \$5,000, 30 percent lose \$10,000, and 20 percent earn \$20,000. What is the expected income from a book?

Solution: We multiply the value of each outcome by the probability of it occurring, and then add the totals:

$$\begin{aligned} 0.25 \cdot \$0 &= \$0 \\ 0.25 \cdot \text{-}\$5{,}000 &= \$1{,}250 \\ 0.30 \cdot \text{-}\$10{,}000 &= \$3{,}000 \\ 0.20 \cdot \$20{,}000 &= \$4{,}000 \end{aligned}$$

Adding all these gives an expected income of -\$250 per book.