July 5, 2006 Math 035 Assignment #2 Solutions

p. 296-299, #6-8, 16(c), 20

6. A single die is cast until it turns up 4. What is the probability that it first turns up 4 on the third throw?

Solution: To have the first 4 appear on the third throw, something besides 4 must appear on each of the first two throws. The probability of this happening on any given throw is $\frac{5}{6}$, so we can use the multiplication rule to find out that

$$\begin{aligned}
\text{P(first 4 is on third roll)} &= \text{P(no 4 on first } \textit{and} \text{ no 4 on second} \\
&\quad \textit{and} \text{ 4 on third)} \\
&= \text{P(no 4 on first)} \cdot \text{P(no 4 on second)} \\
&\quad \cdot \text{P(4 on third)} \\
&= \tfrac{5}{6} \cdot \tfrac{5}{6} \cdot \tfrac{1}{6} = \tfrac{25}{216} \\
&= 0.1157 = 11.57\%
\end{aligned}$$

7. What is the probability of rolling a 5 or a 9 before a 7 with two dice?

Solution: Let A be the event of rolling a 5, 7, or 9, and let B be the event of rolling a 5 or a 9. The question asks for $P(B|A)$, that is, the probability of rolling a 5 or 9 given that one of 5, 7, or 9 has been rolled, since it is on the first roll on which that happens that we will stop and see whether we rolled 5 or 9 first or not.

Looking at our usual table for the roll of two dice, we see that of the 36 total possible outcomes, 4 yield a total of 5, 6 yield a total of 7, and 4 yield a total of 9, so there are 14 outcomes that make event A happen. Of these, $4+4=8$ make event B happen, and since all are equally likely, the basic formula of probability tells us that

$$P(B|A) = \frac{8}{14} = \frac{4}{7} = 0.5714 = 57.14\%$$

8. If Charlie Chaplin spins three gambling wheels, each with the numbers from one to ten, and announces that the first will stop at nine, the second at four, and the third at seven, what is the probability that it will actually happen as he says?

Solution: The three events are independent, so we can use the multiplication rule:

$$\begin{aligned} \text{P(all 3 are correct)} &= \text{P(first is 9 \textit{and} second is 4 \textit{and} third is 7)} \\ &= \text{P(first is 9)} \cdot \text{P(second is 4)} \cdot \text{P(third is 7)} \\ &= \tfrac{1}{10} \cdot \tfrac{1}{10} \cdot \tfrac{1}{10} = \tfrac{1}{1000} \\ &= 0.001 = 0.1\% \end{aligned}$$

16. (c) If I toss two fair coins, what is the probability that they both turn up heads? Both tails? One heads and the other tails?

Solution: The answer is *not* that all three events are equally likely, as you might expect. Instead, we need to construct a table like we did for rolling two dice:

		Second coin	
		H	T
First coin	H	HH	HT
	T	TH	TT

So there are *four* equally likely outcomes. Of these, one corresponds to the event "both turn up heads", one to the event "both turn up tails", and two to the event "one turns up heads and the other tails". Applying the basic formula gives

$$\begin{aligned} \text{P(both are heads)} &= \tfrac{1}{4} = 0.25 = 25\% \\ \text{P(both are tails)} &= \tfrac{1}{4} = 0.25 = 25\% \\ \text{P(one of each)} &= \tfrac{2}{4} = \tfrac{1}{2} = 0.5 = 50\% \end{aligned}$$

20. The result of throwing a fair penny ten times is recorded by a sequence of ten H's and T's. Calculate the probability of each of the following three events: TTTTTTTTTT, TTHTHHTTHT, and TTTTTHHHHH.

Solution: Even though the second of these looks more likely than the other two, the multiplication rule tells us that in fact they all have the same probability. Each throw is independent of the others, and on any given throw, $P(H) = P(T) = \frac{1}{2}$, so we get

$$\begin{aligned}
\mathrm{P(TTTTTTTTTT)} &= \mathrm{P(T)\cdot P(T)\cdot P(T)\cdot P(T)\cdot P(T)} \\
&\quad \mathrm{\cdot\, P(T)\cdot P(T)\cdot P(T)\cdot P(T)\cdot P(T)} \\
&= \tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2} \\
&= (\tfrac{1}{2})^{10} = \tfrac{1}{2^{10}} = \tfrac{1}{1024} = 0.098\% \\
\mathrm{P(TTHTHHTTHT)} &= \mathrm{P(T)\cdot P(T)\cdot P(H)\cdot P(T)\cdot P(H)} \\
&\quad \mathrm{\cdot\, P(H)\cdot P(T)\cdot P(T)\cdot P(H)\cdot P(T)} \\
&= (\tfrac{1}{2})^{10} = \tfrac{1}{2^{10}} = \tfrac{1}{1024} = 0.098\% \\
\mathrm{P(TTTTTHHHHH)} &= \mathrm{P(T)\cdot P(T)\cdot P(T)\cdot P(T)\cdot P(T)} \\
&\quad \mathrm{\cdot\, P(H)\cdot P(H)\cdot P(H)\cdot P(H)\cdot P(H)} \\
&= (\tfrac{1}{2})^{10} = \tfrac{1}{2^{10}} = \tfrac{1}{1024} = 0.098\%
\end{aligned}$$