April 7, 2006 Math 026 Quiz #8 Name: ______________________

The number in brackets gives how many points the question is worth. You have 10 minutes for this quiz; no books, notes, calculators, or rabid dingos are allowed.

(1) 1. Given that the equation $2\sin\theta - \sqrt{3} = 0$ has solutions

$$\theta = \frac{\pi}{3} \quad \text{or} \quad \frac{2\pi}{3}$$

in the interval $0 \leq \theta < 2\pi$, write down an expression for *all* the solutions of the equation. (In other words, I want the formula that uses k.)

Solution:

$$\theta = \frac{\pi}{3} + 2k\pi \quad \text{or} \quad \frac{2\pi}{3} + 2k\pi$$

(4) 2. Solve the equation $2\sin^2\theta - \sin\theta - 1 = 0$ on the interval $0° \leq \theta < 360°$.

Solution: The given quadratic factors as $(2\sin\theta + 1)(\sin\theta - 1) = 0$, so we have

$$\begin{array}{ccc} 2\sin\theta + 1 = 0 & \text{or} & \sin\theta - 1 = 0 \\ \sin\theta = -\frac{1}{2} & \text{or} & \sin\theta = 1 \end{array}$$

In the range $0° \leq \theta < 360°$, the first of these has solutions $\theta = 210°$ and $\theta = 330°$, while the second has solutions $\theta = 90°$. Thus the entire solution set for θ on the given interval consists of

$$90°, \quad 210°, \quad 330°$$

(5) 3. Solve the equation $\tan x - 1 = \sec x$ on the interval $0 \leq x < 2\pi$.

Solution: Putting everything in terms of sine and cosine, we have

$$\frac{\sin x}{\cos x} - 1 = \frac{1}{\cos x}$$

Multiplying both sides by $\cos x$ to clear the denominators gives

$$\sin x - \cos x = 1$$

Squaring both sides yields

$$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$$

and after an application of the Pythagorean identity and the double angle formula for sine, this becomes

$$1 - \sin(2x) = 1$$

which of course is the same as $\sin(2x) = 0$. We want values of x with $0 \leq x < 2\pi$, so multiplying the inequality by two gives $0 \leq 2x < 4\pi$. On this interval, the values of $2x$ that satisfy $\sin(2x) = 0$ are 0, π, 2π, and 3π. Dividing these by two yields four possible answers for x itself:

$$0, \quad \frac{\pi}{2}, \quad \pi, \quad \frac{3\pi}{2}$$

Since we squared both sides of the equation, we must check all four of these answers to see if any of them are extraneous solutions. Upon doing so, we find that the only solution of the original equation is

$$x = \pi$$

Notice that if we had only gone back as far as the equation $\sin x - \cos x = 1$, we would have decided that $x = \frac{\pi}{2}$ was a solution, even though both $\tan x$ and $\sec x$ are undefined at this point, so it cannot be a solution of the original equation. *Thus, we must always go back to the very first equation we were given, to make sure no extraneous solutions have been introduced.*

We could also have done this problem by putting the equation in either of the forms $\sin x = \cos x + 1$ or $\cos x = \sin x - 1$, squaring both sides, using the Pythagorean identity to obtain a quadratic equation in $\sin x$ or $\cos x$, and then proceeding as in number 2. In the end, we would obtain three solutions, two of which would be extraneous, and the other one would be the ones found using our method.

Another approach would be to square both sides in the original equation, giving

$$\tan^2 x - 2\tan x + 1 = \sec^2 x$$

Since one form of the Pythagorean identity states that $\tan^2 x + 1 = \sec^2 x$, this equation reduces to $\tan x = 0$, which has solutions 0 and π. We can check that 0 is extraneous, so the only solution in the given interval is $x = \pi$.