March 24, 2006 Math 026 Quiz #7 Name: ____________________

The number in brackets gives how many points the question is worth. You have 10 minutes for this quiz; no books, notes, calculators, or mariachi bands are allowed.

(2) 1. Write $\cos(6x)\cos(x) + \sin(6x)\sin(x)$ as a single trigonometric function.

Solution: We use the formula for the cosine of the difference of two angles:

$$\cos(A - B) = \cos A \cos B - \sin A \sin B$$

Here $A = 6x$ and $B = x$, so the given expression simplifies to $\cos(6x - x) = \cos x$.

(4) 2. Prove the following trigonometric identity:

$$\frac{\cos x - \sin x}{\cos x + \sin x} = \frac{1 - \sin(2x)}{\cos(2x)}$$

Solution:

$$\begin{aligned}
\frac{\cos x - \sin x}{\cos x + \sin x} &= \frac{\cos x - \sin x}{\cos x + \sin x} \cdot \frac{\cos x - \sin x}{\cos x - \sin x} \\
&= \frac{\cos^2 x - 2\sin x \cos x + \sin^2 x}{\cos^2 x - \sin^2 x} \\
&= \frac{1 - 2\sin x \cos x}{\cos(2x)} \\
&= \frac{1 - \sin(2x)}{\cos(2x)}
\end{aligned}$$

(4) 3. Find the exact value of $\tan(165°)$.

Solution: We use the half-angle formula for tangent:

$$\begin{aligned}
\tan(165°) &= \tan(\frac{330°}{2}) \\
&= \frac{\sin 330°}{1 + \cos 330°} \\
&= \frac{-\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}} = \frac{-1}{2 + \sqrt{3}} \\
&= \frac{-1}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{\sqrt{3} - 2}{2^2 - \sqrt{3}^2} \\
&= \sqrt{3} - 2
\end{aligned}$$

We could have gotten this same answer more quickly by using the alternate half-angle formula for tangent,

$$\tan(\frac{A}{2}) = \frac{1 - \cos A}{\sin A}$$