Solution Key

MATH110 **Quiz6** **Section009**

1 (3 pts) Find $\dfrac{dy}{dx}$ by implicit differentiation.

$$2x^2 + y^2 = xy$$

Solution. Differentiating both sides with respect to x yields

$$\begin{aligned}
\frac{d}{dx}(2x^2 + y^2) &= \frac{d}{dx}(xy) \\
4x + 2y\frac{dy}{dx} &= y + x\frac{dy}{dx} \\
2y\frac{dy}{dx} - x\frac{dy}{dx} &= y - 4x \\
(2y - x)\frac{dy}{dx} &= y - 4x \\
\frac{dy}{dx} &= \frac{y - 4x}{2y - x}.
\end{aligned}$$

2 (3 pts) During the construction of an office building, a hammer is accidentally dropped from a height of 256 ft. The distance (in feet) the hammer falls in t sec is $s = 16t^2$. What is the hammer's velocity when it strikes the ground?
Solution. Solving the quadratic equation $16t^2 = 256$ gives

$$\begin{aligned}
t^2 &= 16 \\
t = 4 \quad &\text{or} \quad t = -4.
\end{aligned}$$

Since t is greater than or equal to 0, $t = 4$. This implies that the hammer hits the ground after 4 seconds. The velcity can be obtained by

$$v(t) = s'(t) = 32t.$$

Thus, the velocity at $t = 4$ becomes $v(4) = 32 \cdot 4 = 128$ ft^2/sec.

3 (4 pts) The volume of a right-circular cylinder of radius r and height h is $V = \pi r^2 h$. Suppose the radius and height of the cylinder are changing with respect to time t.

(a) Find a relationship between $\frac{dV}{dt}$, $\frac{dr}{dt}$ and $\frac{dh}{dt}$.

(b) At a certain instant of time, the radius and height of the cylinder are 2 and 6 in. and are increasing at the rate of 0.1 and 0.3 in./sec, respectively. How fast is the volume of the cylinder increasing?

Solution. For (a). Differentiating both sides of $V = \pi r^2 h$ with respect to t tells us

$$\begin{aligned}
\frac{d}{dt}(V) &= \pi \frac{d}{dt}(r^2 h) \\
\frac{dV}{dt} &= \pi\left(\frac{d}{dt}(r^2) \cdot h + r^2 \frac{d}{dt}(h)\right) \\
\frac{dV}{dt} &= \pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right) \\
\frac{dV}{dt} &= \pi\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right).
\end{aligned}$$

For (b). $r, h, \frac{dr}{dt}$ and $\frac{dh}{dt}$ are given by

$$r = 2, \quad h = 6, \quad \frac{dr}{dt} = 0.1, \quad \frac{dh}{dt} = 0.3$$

Pluggin these values in the equation established in part (a), we have

$$\frac{dV}{dt} = \pi(2 \cdot 2 \cdot 6 \cdot 0.1 + 2^2 \cdot 0.3) = \pi(2.4 + 1.2) = 3.6\pi.$$