

SOLUTION KEY

MATH110

Quiz5

Section009

1 (4 pts) Find the derivative of the function:

$$f(x) = \frac{1}{\sqrt[3]{x^2 - x + 1}}$$

Solution. Using the exponential law, we can express f as follows:

$$f(x) = \frac{1}{\sqrt[3]{x^2 - x + 1}} = (x^2 - x + 1)^{-\frac{1}{3}}.$$

Applying the chain rule yields

$$\begin{aligned} f'(x) &= -\frac{1}{3}(x^2 - x + 1)^{-\frac{1}{3}-1}(2x - 1) = -\frac{1}{3}(2x - 1)(x^2 - x + 1)^{-\frac{4}{3}} \\ &= -\frac{2x - 1}{3\sqrt[3]{(x^2 - x + 1)^4}}. \end{aligned}$$

□

2 (3 pts) The monthly profit in dollars realized from renting x cottages is $p(x) = -x^3 + 4000x - 45,000$. Compute the marginal profit when $x = 15$.
Solution. The marginal profit function is obtained by

$$p'(x) = -3x^2 + 4000.$$

Evaluating $p'(15)$ gives

$$p'(15) = -3 \cdot 15^2 + 4000 = 3325.$$

□

3(3 pts) Find the *second* derivative of the function $y = \sqrt{2x - 1}$.
Solution. y can be expressed in the exponential form as

$$y = \sqrt{2x - 1} = (2x - 1)^{\frac{1}{2}}.$$

Then, the first derivative and the second one are evaluated as follows:

$$y' = \frac{dy}{dx} = \frac{1}{2}(2x - 1)^{\frac{1}{2}-1} \cdot 2 = (2x - 1)^{-\frac{1}{2}}, \quad (1)$$

and

$$y'' = \frac{d^2y}{dx^2} = -\frac{1}{2}(2x - 1)^{-\frac{1}{2}-1} \cdot 2 = -(2x - 1)^{-\frac{3}{2}} = \frac{-1}{\sqrt{(2x - 1)^3}}. \quad (2)$$

□