

SOLUTION KEY

MATH110

Quiz5

Section040

1 (4 pts) Find the derivative of the function:

$$f(x) = \frac{1}{\sqrt[3]{(2x^3 - x + 1)^2}}$$

Solution. Using the exponential law, f can be expressed in the exponential form as follows:

$$f(x) = \frac{1}{\sqrt[3]{(2x^3 - x + 1)^2}} = (2x^3 - x + 1)^{-\frac{2}{3}}.$$

Applying the chain rule yields

$$\begin{aligned} f'(x) &= -\frac{2}{3}(2x^3 - x + 1)^{-\frac{2}{3}-1}(6x^2 - 1) = -\frac{2}{3}(6x^2 - 1)(2x^3 - x + 1)^{-\frac{5}{3}} \\ &= -\frac{2(6x^2 - 1)}{3\sqrt[3]{(2x^3 - x + 1)^5}}. \end{aligned}$$

□

2 (3 pts) The monthly profit in dollars realized from renting x cottages is

$$P(x) = -10x^2 + 1760x - 50,000$$

Compute the marginal profit when $x = 50$.

Solution. The marginal profit function is obtained by

$$P'(x) = -20x + 1760.$$

Evaluating $P'(50)$ gives

$$P'(50) = -20 \cdot 50 + 1760 = 760.$$

□

3(3 pts) Find the *second* derivative of the function $y = \sqrt{2x + 10}$.

Solution. By the exponential law, we can express y in the exponential form as $y = \sqrt{2x + 10} = (2x + 10)^{\frac{1}{2}}$. Then the first and the second derivatives are evaluated:

$$y' = \frac{dy}{dx} = \frac{1}{2}(2x + 10)^{\frac{1}{2}-1} \cdot 2 = (2x + 10)^{-\frac{1}{2}}, \quad (1)$$

and

$$y'' = \frac{d^2y}{dx^2} = -\frac{1}{2}(2x + 10)^{-\frac{1}{2}-1} \cdot 2 = -(2x + 10)^{-\frac{3}{2}} = -\frac{1}{\sqrt{(2x + 10)^3}}. \quad (2)$$

□