Solution Key

MATH110 **Quiz3** **Section040**

1 (2.5 pts) Find the indicated limits.

(a) $\lim_{r\to 9} \frac{\sqrt{r}}{(r-9)^4}$ (b) $\lim_{x\to -\infty} \frac{3x^3-1}{x+5}$

Solution. For (a).

$$\lim_{r\to 9} \frac{\sqrt{r}}{(r-9)^4} = \frac{\sqrt{9}}{(9-9)^4} = \frac{3}{+0} = \infty,$$

or the limit does not exist.

For (b). Dividing both denominator and numerator by x, we have

$$\lim_{x\to -\infty} \frac{3x^3-1}{x+5} = \lim_{x\to -\infty} \frac{3x^2-\frac{1}{x}}{1+\frac{5}{x}} = \frac{3(-\infty)^2-0}{1+0} = \infty,$$

or the limit does not exist.

2 (2.5 pts) Find the indicated limit.

$$\lim_{x\to -3^-} \frac{x^2-9}{|x+3|}$$

Solution. x goes to -3 from the left. This implies that x is less than -3. Thus we get

$$\lim_{x\to -3^-} \frac{x^2-9}{|x+3|} = \lim_{x\to -3^-} \frac{x^2-9}{-(x+3)}.$$

It is of the form $\frac{0}{0}$. Factorizing the numeration, we obtain

$$\lim_{x\to -3^-} \frac{x^2-9}{-(x+3)} = \lim_{x\to -3^-} \frac{(x-3)(x+3)}{-(x+3)} = \lim_{x\to -3^-} \frac{x-3}{-1} = \frac{-3-3}{-1} = 6.$$

3(2.5 pts) Let $f(x) = \begin{cases} 2x^2 & \text{if} \quad x \le 1 \\ \dfrac{x^2-1}{x-1} & \text{if} \quad x > 1. \end{cases}$

(a) Does $\lim_{x \to 1} f(x)$ exist? Explain.

(b) Is f continuous at $x = 1$? Explain.

Solution. For (a). We first evaluate $\lim_{x \to 1^-} f(x)$. Since x goes to 1 from the left, say, x is less than 1, we have $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 2x^2 = 2 \cdot 1^2 = 2$. Now let us evaluate $\lim_{x \to 1^+} f(x)$. Since x goes to 1 from the right, say, x is greater than 1, we get

$$\lim_{x \to 1^+} \frac{x^2-1}{x-1} = \lim_{x \to 1^+} \frac{(x-1)(x+1)}{x-1} = \lim_{x \to 1^+} (x+1) = 1 + 1 = 2.$$

By the definition of the limit, the limit exists and its value is 2.
For (b). The limit of f is already found as $x \to 1$. Let us find the function value of f at $x = 1$. Since $x = 1$ is less than or equal to 1, the function value becomes $f(1) = 2 \cdot 1^2 = 2$. By the definition of continuity, f is continuous at $x = 1$.

4(2.5 pts) Let $f(x) = 2x^2 + x$.

(a) Find the derivative f' of f.

(b) Find an equation of the tangent line to the curve at the point (1,3).

Solution. For (a). By the defintion of the derivative of function, we have

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h)^2 + (x+h) - (2x^2 + x)}{h} \\ &= \lim_{h \to 0} \frac{2(x^2 + 2hx + h^2) + x + h - (2x^2 + x)}{h} \\ &= \lim_{h \to 0} \frac{2x^2 + 4hx + 2h^2 + x + h - 2x^2 - x}{h} \\ &= \lim_{h \to 0} \frac{4hx + 2h^2 + h}{h} \\ &= \lim_{h \to 0} (4x + 2h + 1) \\ &= 4x + 1 \end{aligned}$$

For (a).*Another Solution.* Using the basic rules of differentiations, we have

$$f'(x) = 2 \cdot 2x^{2-1} + 1 = 4x + 1.$$

For (b). The slope of the tangent line is $f'(1) = 4 \cdot 1 + 1 = 5$. By the point-slope formula of the line, we get

$$y - 3 = 5(x - 1),$$

or

$$y = 5x - 2.$$