

Solution key

MATH22

Quiz2

Section002

1 (5 pts) (a) Evaluate the expression and write the result in the form $a + bi$:

i) i^{100}

ii) $\sqrt{-3}\sqrt{-12}$

Solution. For (a).

$$i^{100} = i^{4 \cdot 25} = (i^4)^{25} = 1^{25} = 1.$$

For (b).

$$\sqrt{-3}\sqrt{-12} = (i\sqrt{3})(i\sqrt{12}) = i^2\sqrt{3 \cdot 12} = (-1)\sqrt{36} = -6.$$

(b) Find all solutions of the equation and express them in the form $a + bi$:

$$x^2 - 4x + 5 = 0.$$

Solution. Using the quadratic formula, we have

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm i\sqrt{4}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i.$$

2 (5 pts) Solve $x^{4/3} - 5x^{2/3} + 6 = 0$.

Solution. Let $W = x^{2/3}$. Then $W^2 = x^{4/3}$ and so the equation is of a quadratic type, namely, $W^2 - 5W + 6 = 0$. Solving $W^2 - 5W + 6 = 0$ gives

$$(W - 2)(W - 3) = 0 \tag{1}$$

$$\begin{array}{ll} W = 2 & W = 3 \\ x^{2/3} = 2 & x^{2/3} = 3 \\ x = \pm 2^{3/2} & x = \pm 3^{3/2} \\ x = \pm\sqrt{8} & x = \pm\sqrt{27}. \end{array} \tag{2}$$