

SOLUTION KEY

MATH110

Quiz1

Section040

It consists of 4 questions. Please show all your work to get full credit

1 (2.5 pts) Let f be the function defined by

$$f(x) = \begin{cases} \frac{1}{2}x^2 + 3 & \text{if } x \leq 1 \\ \frac{1}{1-x} & \text{if } x > 1. \end{cases} \quad (1)$$

Find $f(-1)$, $f(0)$, $f(1)$ and $f(2)$.

Solution. Since $x = -1$ is smaller than 1, using $f(x) = \frac{1}{2}x^2 + 3$, we have $f(-1) = \frac{1}{2}(-1)^2 + 3 = \frac{7}{2}$. Also, $f(0) = \frac{1}{2}0^2 + 3 = 3$ and $f(1) = \frac{1}{2}1^2 + 3 = \frac{7}{2}$ since $x = 0$ and $x = 1$ are both smaller than or equal to 1. Last, since $x = 2$ is bigger than 1, using $f(x) = \frac{1}{1-x}$, we get $f(2) = \frac{1}{1-2} = -1$.

2 (2.5 pts) Find the domain of the function.

$$f(x) = \frac{\sqrt{1-x}}{(x-3)(x+4)}.$$

Solution. Since the inside of the square root is greater than or equal to 0, $1-x \geq 0$, say, $x \leq 1$. Also, since the division by 0 is not allowed, $(x-3)(x+4) \neq 0$, which is equivalent to $x \neq 3$ and $x \neq -4$. As a result, the solution set is either

$$\{x \mid x < -4, -4 < x \leq 1\}$$

or

$$(-\infty, -4) \cup (-4, 1].$$

3(2.5pts) Find the composite function $f \circ g$.

$$f(x) = \sqrt{x} + 1; \quad g(x) = x^2 + 1$$

Solution. By the definition of the composite function, we get

$$(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1} + 1.$$

4(2.5pts) Evaluate $h(2)$ where $h = g \circ f$ (Note that you don't have to rationalize the denominator).

$$f(x) = \frac{1}{2x + 1}; \quad g(x) = \sqrt{x + 1}$$

Solution. By the definition of the composite function, we obtain

$$h(2) = (g \circ f)(2) = g(f(2)) = g\left(\frac{1}{5}\right) = \sqrt{\frac{1}{5} + 1} = \sqrt{\frac{6}{5}},$$

since $f(2) = \frac{1}{2 \cdot 2 + 1} = \frac{1}{5}$.