Solution key

MATH22 **Quiz1** **Section002**

1 (2.5 pts) Solve the following equations.
(a)$\dfrac{2}{x-2}+\dfrac{2}{x+2}=\dfrac{8}{x^2-4}$ (b)$7x^{2/3}-28=0.$

Solution. For (a).
Multiplying both sides by the common denominator $(x-2)(x+2)$ gives

$$2(x+2)+2(x-2)=8.$$

$$\begin{aligned} 2x+4+2x-4 &= 8 \\ 4x &= 8 \\ x &= 2. \end{aligned}$$

Let us check our answer. Plugging $x=2$ in the first term of the left hand side of the given equation (a), we see that the denominator is 0. Since the division by 0 is not allowed, the equation has no solution.
For (b).

$$\begin{aligned} 7x^{2/3} &= 28 \\ x^{2/3} &= 4 \\ x &= \pm 4^{3/2} \\ (\text{or} \quad x &= \pm\sqrt{4^3}=\pm\sqrt{64}=\pm 8). \end{aligned} \tag{1}$$

2 (2.5 pts) Solve the following equations.
(a)$2x^2-20x+50=0$ (b)$2x^2-8x+4=0$ (c)$3x^2+2x+2=0.$

Solution. For (a).Using factoring.

$$\begin{aligned} 2(x^2-10x+25) &= 0 \\ 2(x-5)^2 &= 0 \\ x &= 5. \end{aligned}$$

For (b). Using the quadratic formula.

$$\begin{aligned} x &= \frac{-(-8)\pm\sqrt{(-8)^2-4\cdot 2\cdot 4}}{2\cdot 2} \\ x &= \frac{8\pm\sqrt{64-32}}{4} \\ x &= \frac{8\pm 4\sqrt{2}}{4}=2\pm\sqrt{2}. \end{aligned}$$

For (c). Using the quadratic formula.

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{4 - 4 \cdot 3 \cdot 2}}{2 \cdot 3} \\ x &= \frac{-2 \pm \sqrt{-20}}{6} \\ x &= \frac{-2 \pm i2\sqrt{5}}{6} \\ x &= \frac{-1 \pm i\sqrt{5}}{3}. \end{aligned}$$

3(2.5pts) (a) A father is three times as old as his daughter. In 4 years, he will be two times as old as she is. How old is the daughter now? (This question is biologically wrong.)
(b) What quantity of a 50% acid solution must be mixed with a 20% solution to produce 300 mL of 40 % solution?

Solution. For (a).
Let x be the daughter's age now. Then her father's age now is $3x$. In 4 years, her age will be $x + 4$ and his age will be $2(x + 4)$. But her father's age in 4 years is also $3x + 4$. Solving $3x + 4 = 2(x + 4)$ gives $x = 4$. Thus, the daughter's age now is 4 years old.
For (b). Let x be the amount in mL of 50% acid solution to be used. Then $(300 - x)$ mL of 20% solution would have to be used to yield a total of 300 mL of solution. The total amount of pure acid used is $0.50x + 0.20(300 - x) = 0.40(300)$.

$$\begin{aligned} 0.5x + 60 - 0.2x &= 120 \\ 0.3x &= 60 \\ x &= 200. \end{aligned}$$

Thus, 200 mL of 50% solution must be mixed.

4(2.5pts) Use the formula $h = -16t^2 + v_0 t$ discussed in section 1.3. A ball is thrown straight at an initial speed of $v_0 = 40$ft/s
(a) When does the ball reach a height of 24 ft?
(b) When does it reach a height of 48 ft?
(c) What is the greatest height reached by the ball?
(d) When does the ball reach the highest point of its path?
(e) When does the ball hit the ground?.

Solution. For (a). Solving $-16t^2 + 40t = 24$ gives

$$\begin{aligned} -16t^2 + 40t - 24 &= 0 \\ -4(4t^2 - 10t + 6) &= 0 \\ -4(2t - 3)(2t - 2) &= 0. \end{aligned}$$

By zero product property, we get $t = 3/2$ or $t = 1$. The ball reaches the height of 24 ft after 1 sec or 1.5 sec.
For (b). We take a look at $-16t^2 + 40t = 48$. It can be rewritten as $-16t^2 + 40t - 48$. Here, the discriminant $D = b^2 - 4ac = (40)^2 - 4 \cdot (-16) \cdot (-48) = -1472$. The equation has no real solution. This means that the ball never reach the height of 48 ft.
For (c). Let us look at $h = -16t^2 + 40t$. We now move all terms to one side of the equal sign, namely, $-16t^2 + 40t - h = 0$. Since the ball reach only once the highest height, the discriminant $D = 0$, i.e.,

$$0 = D = 40^2 - 4 \cdot (-16) \cdot h,$$

which tells us that $h = 25$.
For (d). From (c), we obtain the equation $-16t^2 + 40t = 25$. Solving the quadratic equation, we have

$$\begin{aligned} -16t^2 + 40t - 25 &= 0 \\ -(16t^2 - 40t + 25) &= 0 \\ -(4t - 5)^2 &= 0 \\ t &= 5/4. \end{aligned}$$

Thus, the ball reaches its highest height.
For (e). Hitting the ground says $h = 0$. We only have to solve $-16t^2 + 40t = 0$.

$$\begin{aligned} -8t(2t - 5) &= 0 \\ t = 0 \quad \text{or} \quad & t = 5/2. \end{aligned}$$

Therefore, the ball hits the ground in $5/2$ seconds.