

# Instructor Solution.

MATH110

Quiz13

Section009

Name:

~~It consists of 4 questions. Please show all your work to get full credit.~~

1 (2.5 pts) Find the indefinite integral.

$$\int \frac{x}{3x^2-1} dx$$

Sol set  $u = 3x^2 - 1$ ,  $du = 6x dx \Rightarrow \frac{1}{6} du = x dx$

$$\begin{aligned} \int \frac{x}{3x^2-1} dx &= \int \frac{1}{3x^2-1} x dx = \int \frac{1}{u} \frac{1}{6} du = \frac{1}{6} \int \frac{1}{u} du \\ &= \frac{1}{6} \ln|u| + c = \frac{1}{6} \ln|3x^2-1| + c. \end{aligned}$$

2 (2.5 pts) Find the indefinite integral.

$$\int \frac{1}{x \ln x} dx$$

Sol set  $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$$\begin{aligned} \int \frac{1}{x \ln x} dx &= \int \frac{1}{\ln x} \cdot \frac{1}{x} dx = \int \frac{1}{u} du \\ &= \ln|u| + c = \ln|\ln x| + c. \end{aligned}$$

3 (2.5 pts) Find the indefinite integral.

$$\int x(x-1)^5 dx$$

Sol Set  $u = x-1 \Rightarrow du = dx$  &  $x = u+1$

$$\begin{aligned}\int x(x-1)^5 dx &= \int (u+1)u^5 du = \int (u^6 + u^5) du \\ &= \frac{1}{7}u^7 + \frac{1}{6}u^6 + C = \frac{1}{7}(x-1)^7 + \frac{1}{6}(x-1)^6 + C.\end{aligned}$$

4 (2.5 pts) Find the function  $f$  given that the slope of the tangent line to the graph of  $f$  at any point  $(x, f(x))$  is  $f'(x)$  and that the graph of  $f$  passes through the given point.

$$f'(x) = -2xe^{-x^2+1}; (1, 0)$$

Sol

$$f(x) = \int -2x \cdot e^{-x^2+1} dx$$

Set  $u = -x^2+1 \Rightarrow du = -2x dx$

$$\begin{aligned}f(x) &= \int -2x \cdot e^{-x^2+1} dx = \int e^{-x^2+1} \cdot (-2x) dx \\ &= \int e^u du = e^u + C \\ &= e^{-x^2+1} + C\end{aligned}$$

Since  $f(1) = 0$ ,

$$0 = f(1) = e^{-1^2+1} + C = 1 + C$$

$$\Rightarrow C = -1 \quad \therefore f(x) = e^{-x^2+1} - 1.$$