

Name: _____

~~It consists of 4 questions. Please show all your work to get full credit.~~

1 (2.5 pts) Find the indefinite integral.

$$\int \left(e^{-2x} + \frac{1}{x^2} + \frac{3}{x} \right) dx$$

Sol

$$= -\frac{1}{2}e^{-2x} + \int x^{-2} dx + 3 \int \frac{1}{x} dx$$

$$= -\frac{1}{2}e^{-2x} + \frac{1}{-2+1} x^{-2+1} + 3 \ln|x| + C$$

$$= -\frac{1}{2}e^{-2x} - \frac{1}{x} + 3 \ln|x| + C$$

2 (2.5 pts) Find the indefinite integral.

$$\int \left(\frac{u^3 + 2u^2 - u}{3u} \right) du$$

Sol

$$= \frac{1}{3} \int u^2 du + \frac{2}{3} \int u du - \frac{1}{3} \int 1 du$$

$$= \frac{1}{3} \cdot \frac{1}{2+1} u^{2+1} + \frac{2}{3} \cdot \frac{1}{1+1} u^{1+1} - \frac{1}{3} u + C$$

$$= \frac{1}{9} u^3 + \frac{1}{3} u^2 - \frac{1}{3} u + C$$

3 (2.5 pts) If $f'(x) = 1 + e^x + \frac{1}{x}$ and $f(1) = 3 + e$, what is f ?

Sol

$$f(x) = \int \left(1 + e^x + \frac{1}{x}\right) dx = x + e^x + \ln|x| + C$$

$$f(1) = 1 + e^1 + \ln|1| + C = 1 + e + C$$

Since $f(1) = 3 + e$, $3 + e = 1 + e + C \Rightarrow C = 2$.

$$\therefore f(x) = x + e^x + \ln|x| + 2.$$

4 (2.5 pts) The velocity of a car (in feet/second) t sec after starting from rest is given by the function

$$f(t) = 2\sqrt{t} \quad (0 \leq t \leq 30)$$

Find the car's position at any time t .

Sol

$$S(t) = \int 2\sqrt{t} dt = \int 2t^{\frac{1}{2}} dt = 2 \cdot \frac{1}{\frac{1}{2}+1} t^{\frac{1}{2}+1} + C$$
$$= 2 \cdot \frac{1}{\frac{3}{2}} t^{\frac{3}{2}} + C = \frac{4}{3} t^{\frac{3}{2}} + C$$

Since $S(0) = 0$, $C = 0$.

$$\therefore S(t) = \frac{4}{3} t^{\frac{3}{2}}$$