

Name:

~~It consists of 4 questions. Please show all your work to get full credit~~

1 (2.5 pts) Find the indefinite integral.

$$\int \left( e^{-2x} + \frac{1}{x^2} + \frac{3}{x} \right) dx$$

(Sol)

$$= -\frac{1}{2}e^{-2x} + \frac{1}{-2+1}x^{-2+1} + 3\ln|x| + c$$

$$= -\frac{1}{2}e^{-2x} - \frac{1}{x} + 3\ln|x| + c$$

2 (2.5 pts) Find the indefinite integral.

$$\int \left( \frac{t^3 + \sqrt[3]{t}}{t^2} \right) dt$$

$$\underline{\text{Sol}} = \int t + t^{-\frac{5}{3}} dt = \int t dt + \int t^{-\frac{5}{3}} dt$$

$$= \frac{1}{1+1}t^{1+1} + \frac{1}{-\frac{5}{3}+1}t^{-\frac{5}{3}+1} + c$$

$$= \frac{1}{2}t^2 - \frac{3}{2}t^{-\frac{2}{3}} + c. \quad (\text{or } = \frac{1}{2}t^2 - \frac{3}{2\sqrt[3]{t^2}} + c)$$

3 (2.5 pts) If  $f'(x) = 1 + e^x + \frac{1}{x}$  and  $f(1) = 3 + e$ , what is  $f$ ?

$$\text{Sol } f(x) = \int \left(1 + e^x + \frac{1}{x}\right) dx = x + e^x + \ln|x| + C$$

$$f(1) = 1 + e^1 + \ln|1| + C = 1 + e + C$$

$$\text{Since } f(1) = 3 + e, \quad C = 2.$$

$$\therefore f(x) = x + e^x + \ln|x| + 2.$$

4 (2.5 pts) The velocity (in feet/second) of a maglev is

$$v(t) = 2\sqrt{t} + 3 \quad (0 \leq t \leq 120)$$

At  $t = 0$ , it is at the station. Find the function giving the position of the maglev at time  $t$ , assuming that the motion takes place along a straight stretch of track.

$$\begin{aligned} \text{Sol } S(t) &= \int (2\sqrt{t} + 3) dt = 2 \cdot \frac{1}{\frac{1}{2} + 1} t^{\frac{1}{2} + 1} + 3t + C \\ &= \frac{4}{3} t^{\frac{3}{2}} + 3t + C. \end{aligned}$$

$$\text{Since } S(0) = 0, \quad C = 0 \quad \therefore S(t) = \frac{4}{3} t^{\frac{3}{2}} + 3t.$$