Instructor Solution

**MATH110** **Quiz11** **Section040**
**Name:**

It consists of 3 questions. *Please show all your work to get full credit*
**1 (5 pts)** Find the **second** derivatve of the function.

(a) $f(x) = e^{-4x} + 2e^{3x}$ (b) $f(x) = \ln x^2$

Sol

(a) $f'(x) = -4e^{-4x} + 6e^{3x}$

$f''(x) = 16e^{-4x} + 18e^{3x}$

(b) $f(x) = 2\ln x$

$f'(x) = \frac{2}{x}$

$f''(x) = -\frac{2}{x^2}$

**2 (2 pts)** The radioactive element polonium decays according to the law

$$Q(t) = Q_0 \cdot 2^{-(t/140)}$$

where $Q_0$ is the initial amount and the time $t$ is measured in days. If the amount of polonium left after 280 days is 20 mg, what was the initial amount present?

Sol

$$20 = Q_0 \cdot 2^{-\frac{280}{140}}$$

$$\Rightarrow 20 = Q_0 2^{-2} = \frac{Q_0}{4}$$

$$\Rightarrow Q_0 = 80 \text{ mg}$$

**3 (3 pts)** The growth rate of the bacterium *Escherichia coli*, a common bacterium found in the human intestine, is proportional to its size. Under ideal laboratory conditions, when this bacterium is grown in a nutrient broth medium, the number of cells in a culture doubles approximately every 20 min.

**a.** If the initial cell population is 100, determine the function $Q(t)$ that expresses the exponential growth of the number of cells of this bacterium as a function of time $t$ (in minutes).

**b.** How long will it take for a colony of 100 cells to increase to a population of 1 million?

Sol

(a). $Q(t) = 100e^{kt}$

$200 = 100e^{k \cdot 20}$

$2 = e^{20k}$

$\ln 2 = 20k$

$\therefore k = \frac{\ln 2}{20}.$

(b) $1{,}000{,}000 = 100e^{\frac{\ln 2}{20} \cdot t}$

$10{,}000 = e^{\frac{\ln 2}{20} \cdot t}$

$\ln(10{,}000) = \frac{\ln 2}{20} t$

$\therefore t = \frac{20 \ln(10{,}000)}{\ln 2}$