Instructor Solution.

**MATH110** **Quiz10** **Section009**

~~**Name:**~~

~~It consists of 4 questions.~~ *Please show all your work to get full credit*

**1 (2.5 pts)** Find the interest rate needed for an investment of \$4000 to double in 5 yr if interest is compounded continuously.

Sol $A = Pe^{rt}$

$8000 = 4000e^{r5} \Rightarrow 8000 = 4000e^{5r}$

$\Rightarrow 2 = e^{5r}$

$\Rightarrow \ln 2 = \ln e^{5r}$

$\Rightarrow \ln 2 = (5r)\cdot \ln e$

$\Rightarrow \ln 2 = 5r$

$\Rightarrow \frac{\ln 2}{5} = r.$

**2 (2.5 pts)** Find the derivative of the function.

$$f(x) = \frac{e^x - 1}{e^x + 1}$$

Sol

$$f'(x) = \frac{(e^x+1)(e^x-1)' - (e^x-1)(e^x+1)'}{(e^x+1)^2}$$

$$= \frac{(e^x+1)\cdot e^x - (e^x-1)e^x}{(e^x+1)^2}$$

$$= \frac{e^{2x}+e^x - (e^{2x}-e^x)}{(e^x+1)^2}$$

$$= \frac{e^{2x}+e^x - e^{2x}+e^x}{(e^x+1)^2}$$

$$= \frac{2e^x}{(e^x+1)^2}$$

**3 (2.5 pts)** Find an equation of the tangent line to the graph of $y = x \ln x$ at the point (1,0).

Sol $y - 0 = m(x-1)$.

Find $m = f'(1)$.

$f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$f'(1) = \ln 1 + 1 = 0 + 1 = 1$.

$\therefore y = 1(x-1) \Rightarrow y = x - 1$.

**4 (2.5 pts)** Use logarithmic differentiation to finc the derivative of the function.

$$y = \frac{(2x^2-1)^5}{\sqrt{x+1}}.$$

Sol $\ln y = \ln\left(\frac{(2x^2-1)^5}{\sqrt{x+1}}\right) = \ln(2x^2-1)^5 - \ln(\sqrt{x+1})$

$= 5\ln(2x^2-1) - \frac{1}{2}\ln(x+1)$

$\Rightarrow \frac{y'}{y} = 5 \cdot \frac{4x}{2x^2-1} - \frac{1}{2} \cdot \frac{1}{x+1}$

$\Rightarrow \frac{y'}{y} = \frac{20x}{2x^2-1} - \frac{1}{2(x+1)}$

$\Rightarrow y' = \left(\frac{20x}{2x^2-1} - \frac{1}{2(x+1)}\right) \cdot y$

$\Rightarrow y' = \left(\frac{20x}{2x^2-1} - \frac{1}{2(x+1)}\right) \cdot \frac{(2x^2-1)^5}{\sqrt{x+1}}$.