The continuum mechanics of turbulence: a generalized Navier–Stokes-α equation with complete boundary conditions

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Abstract

The direct numerical simulation of turbulence at Reynolds numbers in excess of a few thousand provides a formidible computational problem, even with access to state-of-the-art supercomputers. For this reason, there remains a strong interest in alternative methods that resolve only large-scale motions while modeling small-scale motions via filtering, the most well-known of which are large eddy simulation and closure approximations based on Reynolds averaged equations. While they reduce computational costs, the additional dissipation associated with these methods can lead to artifically sluggish flows. A method that avoids this difficulty is provided by simulations based on an equation—known as the Navier–Stokes-α (NS-α) equation—obtained by Lagrangian averaging. Aside from the density ρ and the shear viscosity μ of the fluid, the NS-α equation involves an additional material parameter $\alpha > 0$ carrying dimensions of length. In the context of Lagrangian averaging, α is the statistical correlation length of the excursions taken by a fluid particle away from its phase-averaged trajectory. More intuitively, α can be viewed as the characteristic length of the smallest eddies that the model is capable of resolving.

In this talk, we use the framework of Fried & Gurtin (2005) to develop an alternative continuum-mechanical formulation leading to a generalization of the NS-α equation. That generalization involves two additional material length scales, one of energetic origin and the other of dissipative origin. The NS-α equation arises on equating these length scales. We explore the impact of these length scales on the energy spectrum and inertial range.

In contrast to Lagrangian averaging, our formulation also delivers boundary conditions and a complete thermodynamic framework. The boundary conditions also involve one more material length scale.

As an application, we consider the classical problem of turbulent flow in a plane, rectangular channel with fixed, impermeable, slip-free walls and make comparisons with results obtained by direct numerical simulations. When the additional material parameter associated with the boundary conditions is signed to ensure satisfaction of the second law (of thermodynamics) at the channel walls the theory delivers solutions that agree neither quantitatively nor qualitatively with observed features of turbulent plane channel flow. On the contrary, excellent agreement arises when the sign of the additional material parameter associated with the boundary conditions violates the second law.

Although Marsden & Shkoller (2001) recently established well-posedness results for the NS-α equation on bounded domains, their analysis is predicated upon on thermodynamically stable boundary conditions and therefore cannot pertain to turbulent flows. The question of whether initial-boundary-value problems for the NS-α equations are well-posed when boundary conditions appropriate to turbulence are imposed therefore remains open. An additional question of central importance concerns whether solutions to initial-boundary-value problems for the NS-α equations converge to solutions of initial-boundary-value problems for the Navier–Stokes equations. Because of the nonstandard thermodynamic structure of the theory, it seems very likely that answers to these questions will require novel analytical approaches.

References

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