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Acoustic bandgap formation in a periodic structure with multilayer unit cells

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Abstract. Using the transfer matrix method and Floquet's theorem we have derived the dispersion relation for acoustic wave propagation in a periodic layered structure in which each unit cell contains several sub-layers of different materials. Structures with unit cells containing two, four and six sublayers of different thickness and two types of materials were used as examples for our study. It was found that narrow passbands and broad stopbands could be obtained when the unit cell has more than two sublayers. The calculated results were verified experimentally using glass–water structures containing only one to three cells. Good agreement was obtained between the experimental results and the transfer matrix calculations. Desired bandgap structures can be produced in less than three cells, revealing application potential for the bandgap materials in vibration control devices and acoustic filters.

1. Introduction

In recent years, photonic bandgap materials have been widely studied [1–3]. These materials are made of periodic arrays of two transparent dielectrics. Such a structure can produce stopbands in which electromagnetic waves of certain frequencies cannot propagate, which is analogous to the Bragg reflection of electrons in solids. The study of photonic crystals arouses the interest in phononic (acoustic) bandgap structures because the nature of wave propagation is the same.

Periodic structures exhibiting acoustic bandgaps were predicted theoretically [4–9]. Several theoretical methods have been developed to study the bandgap phenomena, such as the transfer matrix method [6], plane-wave method [7, 8] and effective medium method [9]. There are also a few experimental investigations reported in the literature on this topic [10–12]. The existence of acoustic bandgaps is of interest for many applications, such as elastic/acoustic wave filters, ultrasonic silent blocks (or acoustic passband mirrors), and the concept can also help to improve the efficiency of ultrasonic transducers [13, 14].

All previously reports on wave propagation in periodic acoustic media were focused on layered structures with only two sublayers in each unit cell. In this paper, we use the transfer matrix method and Floquet's theorem to derive the dispersion relations of acoustic waves in layered structures in which each unit cell contains several sublayers of different materials with different thickness. The periodic structures made of two, four and six sublayers of two different materials were selected as demonstration cases for easy comparison with experiments. For finite size periodic structures, Floquet's theorem does not apply and the transmission coefficient spectrum was calculated using the transfer matrix technique; the numerical results were verified by experiments for a system containing only three cells.

2. Dispersion relation for infinite periodic layered structures

Assuming an infinite periodic layered structure in which each unit cell contains M sublayers. The material and thickness of these sublayers are all different. Figure 1 illustrates a unit cell of the infinite periodic layered structure under study. When an acoustic plane wave is incident normal into the structure, we can write the wavefunction inside the mth layer of the nth cell in the following form:

$$\psi_{n,m} = a_{n,m}\,\mathrm{e}^{\mathrm{i}(2\pi f t - k_m x)} + b_{n,m}\,\mathrm{e}^{\mathrm{i}(2\pi f t + k_m x)} \qquad (1)$$

where n is the unit cell number and m is the layer number within the nth cell. The first and the second terms on the right-hand side of equation (1) represent the forward and the backward waves, respectively; $k_m = 2\pi f/c_m$ is the wavevector for waves propagating in material m; f is the wave frequency; and c_m is the phase velocity of the acoustic wave in the mth layer.

The continuity requirements for the wavefunction and the stress at the interface between the sublayers in the nth unit cell lead to the following relations among the coefficients $a_{n,j}$, $b_{n,j}$ and $a_{n+1,j}$, $b_{n+1,j}$ [6, 15]

$$\begin{pmatrix} a_{n,j} \\ b_{n,j} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_{n+1,j} \\ b_{n+1,j} \end{pmatrix} \qquad (2)$$

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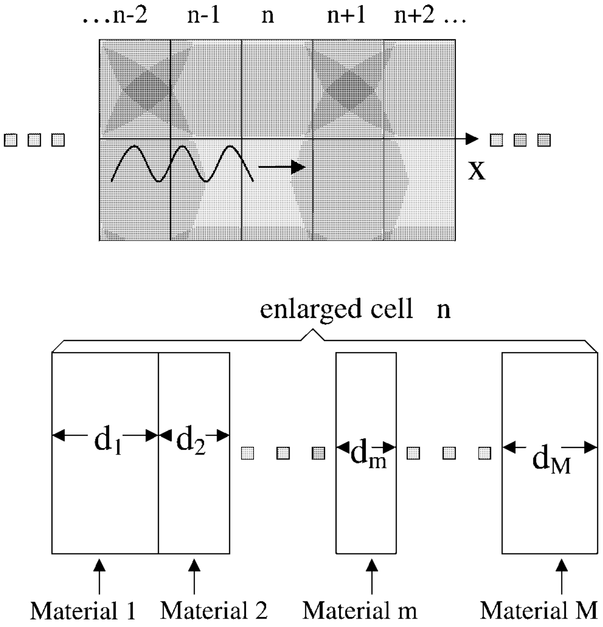

Figure 1. A one-dimensional layered structure with each unit cell consisting of M sublayers of different materials with different thickness.

where the (2 × 2) transfer matrix is given by

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \prod_{m=1}^{M} \begin{pmatrix} A_m & B_m \\ C_m & D_m \end{pmatrix} \tag{3}$$

with

$$A_m = \frac{z_m + z_{m+1}}{2z_m} \mathrm{e}^{\mathrm{i}(k_m - k_{m+1})x_m}$$

$$B_m = \frac{z_m - z_{m+1}}{2z_m} \mathrm{e}^{\mathrm{i}(k_m - k_{m+1})x_m}.$$

In equation (3) C_m and D_m are the complex conjugates of B_m and A_m respectively, $z_m = \rho_m v_m$ is the acoustic impedance with ρ_m and v_m the density and velocity of the mth sublayer, respectively, and x_m is the coordinate of the interface between the mth and $(m + 1)$th sublayers. Equation (3) includes the acoustic properties of all layers in one unit cell.

According to Floquet's theorem, for an infinite one-dimensional periodic system, the coefficients $a_{n,j}$ and $b_{n,j}$ for the same material in different unit cells must be the same except a phase shift, i.e.,

$$\begin{pmatrix} a_{n,j} \\ b_{n,j} \end{pmatrix} = \begin{pmatrix} a_{n+1,j} \\ b_{n+1,j} \end{pmatrix} \mathrm{e}^{-\mathrm{i}kd} \tag{4}$$

where k is the effective wavevector for waves propagating in the whole structure and d is the unit cell size (period).

Using (2) and (4) we can derive the dispersion relation for the whole structure,

$$k = \frac{1}{d} \arccos\left(\frac{1}{2}(A + D)\right). \tag{5}$$

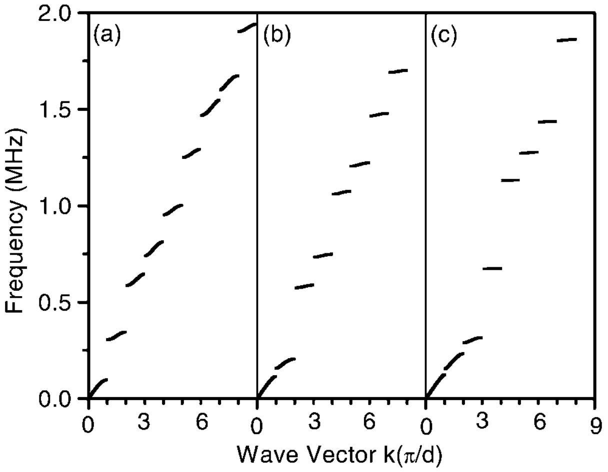

Figure 2. Dispersion relations for infinite multilayer periodic structures: (a) two sublayers in each unit cell, $d_1 = 3.690$ mm (glass) and $d_2 = 2.310$ mm (water); (b) four sublayers in each unit cell, $d_1 = 1.230$ mm (glass), $d_2 = 1.010$ mm (water), $d_3 = 2.460$ mm (glass) and $d_4 = 1.300$ mm (water); (c) six sublayers in each unit cell, $d_1 = 1.230$ mm (glass), $d_2 = 0.650$ mm (water), $d_3 = 1.230$ mm (glass) and $d_4 = 1.155$ mm (water), $d_5 = 1.230$ mm (glass), $d_6 = 0.505$ mm (water).

Equation (5) has been obtained in periodic layered structures with two sublayers in each unit cell [15]. For the multiplayer unit cell situation, the quantity $(A + D)$ in (5) can be obtained directly from (3):

$$\begin{aligned} A + D = {} & \prod_{h=1}^{M} \cos(k_h d_h) - \sum_{h=1}^{M-1} \sum_{j=h+1}^{M} \left(\frac{z_h}{z_j} + \frac{z_j}{z_h}\right) \\ & \times \left[\prod_{k=h,j} \sin(k_k d_k)\right] \prod_{l(\neq h,j)=1}^{M} \cos(k_l d_l) \\ & + \sum_{h=1}^{M-3} \sum_{j=h+2}^{M-1} \sum_{k=h+1}^{j-1} \sum_{l=j+1}^{M} \left(\frac{z_h z_j}{z_l z_k} + \frac{z_l z_k}{z_h z_j}\right) \\ & \times \left[\prod_{m=h,j,k,l} \sin(k_m d_m)\right]\left[\prod_{n(\neq h,j,k,l)=1}^{M} \cos(k_n d_n)\right] \\ & - \sum_{h=1}^{M-5} \sum_{j=h+2}^{M-3} \sum_{k=h+4}^{M-1} \sum_{l=h+1}^{j-1} \sum_{m=j+1}^{k-1} \sum_{n=k+1}^{M} \left(\frac{z_h z_j z_k}{z_l z_m z_n} + \frac{z_l z_m z_n}{z_h z_j z_k}\right) \\ & \times \left[\prod_{p=h,j,k,l,m,n} \sin(k_p d_p)\right] \prod_{q(\neq h,j,k,l,m,n)=1}^{M} \cos(k_q d_q) \\ & + \sum_{h=1}^{M-7} \sum_{j=h+2}^{M-5} \sum_{k=h+4}^{M-3} \sum_{l=h+6}^{M-1} \sum_{m=h+1}^{j-1} \sum_{n=j+1}^{k-1} \sum_{p=k+1}^{l-1} \sum_{q=l+1}^{M} \\ & \times \left(\frac{z_h z_j z_k z_l}{z_m z_n z_p z_q} + \frac{z_m z_n z_p z_q}{z_h z_j z_k z_l}\right) \left[\prod_{r=h,j,k,l,m,n,p,q} \sin(k_r d_r)\right] \\ & \times \prod_{s(\neq h,j,k,l,m,n,p,q)=1}^{M} \cos(k_s d_s) - \cdots \end{aligned} \tag{6}$$

where the subscripts h, j, k, l, m, n, p, q, r and s are all integers. When there are two, four and six sublayers in a unit cell, (6) is truncated at the second, third and fourth terms, respectively, and the specific forms of $(A + D)$ are given in the appendix.

In figure 2, we show the calculated dispersion relations for acoustic waves propagating through infinite periodic layered structures with each unit cell containing two, four and six sublayers. The sublayers were made of two materials with different thickness and arranged alternately within the unit cell. The properties of the two materials are: (1) glass with a density of 2459 kg m^{-3} and a phase velocity of 5660 m s^{-1} and (2) distilled water with a density 1000 kg m^{-3} and a phase velocity of 1480 m s^{-1}. In each case, the volume ratio between the glass and the water was kept constant, and the cell period was kept at $d = 6.0$ mm. Figure 2(a) is a typical dispersion relation for an infinite periodic one-dimensional structure formed by unit cells of two sublayers. The piecewise dispersion curves represent the passbands while the gaps are the stopbands. If the unit cell contains four sublayers of water and glass, narrow passbands are produced, as shown in figure 2(b), except the first two passbands. Extremely narrow passbands appear if the unit cell is divided into six different sublayers as shown in figure 2(c). Many passbands become almost a straight line perpendicular to the frequency axis. Although the number of passbands for the cases shown in figures 2(a)–(c) remains unchanged in the frequency range 0–2 MHz, the bandwidth ratio between the stopbands and passbands increases significantly from 2.69 for the case in figure 2(a) to 6.08 for the case in figure 2(b) and to 6.97 for the case in figure 2(c). In other words, bandgaps become much wider when we subdivide the unit cell into non-equivalent sublayers. The only exceptions are the first passband and the first stopband, they become wider and narrower, respectively, when the unit cell is divided into more sublayers.

The Floquet condition equation (4) is valid only for infinite systems, which does not reflect the experimental situation since the measured structures are always finite in size. For a finite system, the dispersion relation can be obtained by the transfer matrix technique defined in [6]: Band structures are not fully developed for a finite system; therefore, it is more meaningful to look at the frequency spectrum of the transmission coefficient.

3. Frequency spectrum of the transmission coefficient of multi-sublayer periodic structures of finite size

We select glass and water as the base material for convenience in experimental testing. A dissipation factor was introduced in the calculations through adding a small imaginary component to the phase velocity in glass, i.e. $c_g = (5660 + 10\mathrm{i})$ m s^{-1}. Each test structure contains only one to three cells with each cell contains two, four and six sublayers of glass and water in an alternating arrangement. The transfer matrix technique defined in [6] for a finite system was used to calculate the acoustic band structure.

The experiments were conducted in a water tank using a set-up similar to that described in [12, 13]. Two broadband ultrasonic transducers with centre frequency of 1.5 MHz were used to cover the frequency range of 0.5–2 MHz, one as the transmitter and the other as the receiver. The nominal active diameter of the transducers is 12 mm. The transmitting transducer was driven by a DPR35 Pulser/Receiver and the received signal was fed to a digital oscilloscope (Tektronix TDS 460A with fast Fourier transform analysis capabilities).

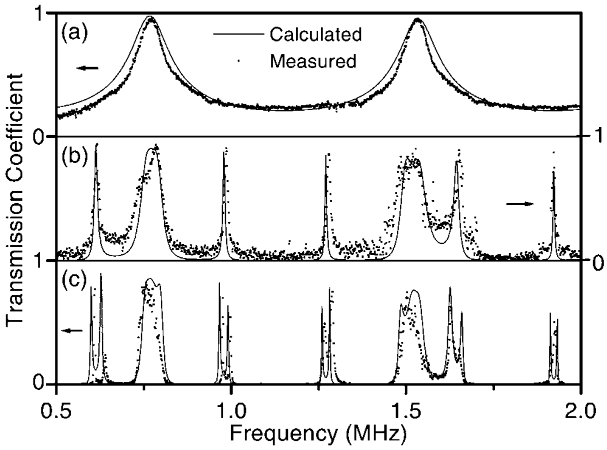

Figure 3. Calculated and measured transmission coefficients as functions of frequency for finite periodic structures containing two sublayers in the unit cell. The thickness values of the two sublayers are the same as those in figure 2(a): (a) a structure with only one unit cell, (b) a structure with two unit cells and (c) a structure with three unit cells.

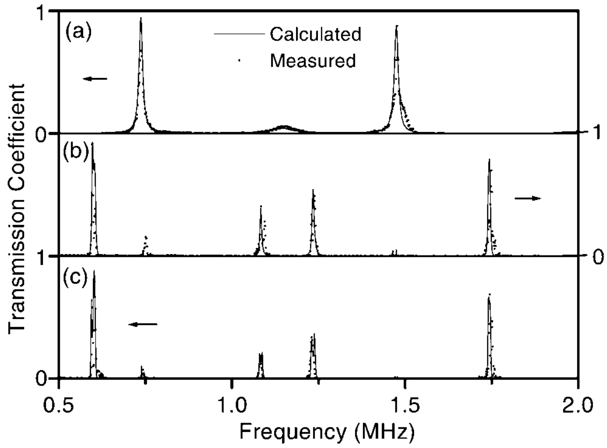

Figure 4. Calculated and measured transmission coefficients as functions of frequency for finite structures containing four sublayers in the unit cell. The thickness values of the four sublayers are the same as those in figure 2(b): (a) a structure with only one unit cell, (b) a structure with two unit cells and (c) a structure with three unit cells.

A ten-signal average scheme was used to improve the signal-to-noise ratio and the reference signal of water was subtracted in the data processing stage.

Figure 3 shows the calculated and measured frequency spectra of the transmission coefficient for structures consisting of one, two and three cells with each unit cell containing only one layer of glass and one layer of water. Good agreement was obtained between the measured and calculated spectra of the transmission coefficient. Gibbs-type oscillations are seen in the passband for the case shown in figure 3(c), which is consistent with that of [12].

Figure 4 is for the case with four sublayer unit cells, i.e., two layers of water and two layers of glass arranged alternately in the cell. The unit cell size and the volume ratio of the two types of materials were kept the same as the case shown in figure 3. It is interesting to note that a band structure with two narrow passbands can be seen with only one unit

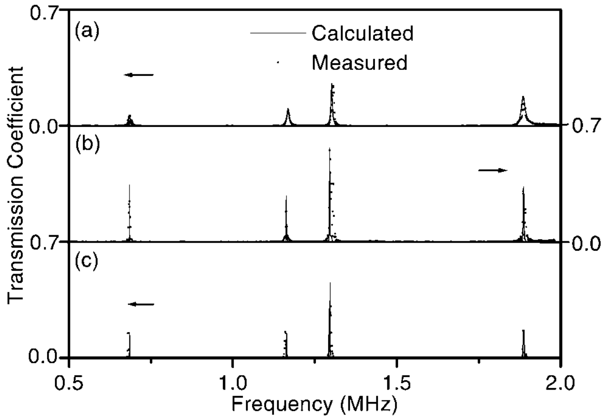

Figure 5. Calculated and measured transmission coefficients as functions of frequency for finite structures containing six sublayers in the unit cell. The thickness values of the six sublayers are the same as those in figure 2(c): (a) a structure with only one cell, (b) a structure with two cells and (c) a structure with three unit cells.

cell. The results also suggest that two unit cells appear to be enough to provide well defined bandgaps (figure 4(b)). This is a significant finding because the total thickness of the structure allowed in real applications is always limited.

The band structure of the six sublayers unit cell systems (i.e. three layers of water and three layers of glass) is shown in figure 5. The period and the volume ratio of the two types of materials were kept the same as for the cases shown in figures 3 and 4. Narrower passbands than those of figures 3 and 4 were obtained in the frequency range of 0.5–2 MHz, which is consistent with the dispersion relation for an infinite system (figure 2(c)). The predicted small peak centred at about 1.38 MHz has a very weak intensity and could not be detected in the experiments.

4. Conclusions

Acoustic bandgaps produced by periodic structures originate from multiple reflections at the interface due to the impedance mismatch between the adjacent materials. In this investigation, we subdivided the unit cell into several different sublayers, which can alter the band structures. Using the transfer matrix method and Floquet's theorem, the dispersion relations were derived for infinite periodic structures with the unit cell containing arbitrary number of sublayers made of different materials with different thickness. Numerical calculations were performed for the cases with unit cells containing two, four and six sublayers made of glass and water. The results showed that narrow passbands are produced when the unit cell is subdivided into more than four sublayers. The ratio of the total stopband width versus passband width increases significantly from the case of two sublayer cells to the case of six sublayer cells. These theoretical results were confirmed by the measurements of the frequency spectrum of the transmission coefficient for several finite structures. Our results showed that it is possible to produce high-quality acoustic filters with very narrow passbands using less than three cells when the unit cell is subdivided into more than four layers.

Acknowledgments

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Appendix

The expression of $(A + D)$ in equation (6) for different numbers of sublayers in one unit cell.

(1) Each cell contains two sublayers:

$$\begin{aligned} A + D &= \cos(k_1d_1)\cos(k_2d_2) \\ &\quad +\frac{1}{2}\left(\frac{z_1}{z_2}+\frac{z_2}{z_1}\right)\sin(k_1d_1)\sin(k_2d_2). \end{aligned} \tag{A1}$$

(2) Each cell contains four sublayers:

$$\begin{aligned} A + D &= \cos(k_1d_1)\cos(k_2d_2)\cos(k_3d_3)\cos(k_4d_4) \\ &\quad -\frac{1}{2}\left(\frac{z_1}{z_2}+\frac{z_2}{z_1}\right)\sin(k_1d_1)\sin(k_2d_2)\cos(k_3d_3)\cos(k_4d_4) \\ &\quad -\frac{1}{2}\left(\frac{z_2}{z_3}+\frac{z_3}{z_2}\right)\sin(k_2d_2)\sin(k_3d_3)\cos(k_1d_1)\cos(k_4d_4) \\ &\quad -\frac{1}{2}\left(\frac{z_3}{z_4}+\frac{z_4}{z_3}\right)\sin(k_3d_3)\sin(k_4d_4)\cos(k_1d_1)\cos(k_2d_2) \\ &\quad -\frac{1}{2}\left(\frac{z_1}{z_3}+\frac{z_3}{z_1}\right)\sin(k_1d_1)\sin(k_3d_3)\cos(k_2d_2)\cos(k_4d_4) \\ &\quad -\frac{1}{2}\left(\frac{z_2}{z_4}+\frac{z_4}{z_2}\right)\sin(k_2d_2)\sin(k_4d_4)\cos(k_1d_1)\cos(k_3d_3) \\ &\quad +\frac{1}{2}\left(\frac{z_1z_3}{z_2z_4}+\frac{z_2z_4}{z_1z_3}\right)\sin(k_1d_1)\sin(k_2d_2) \\ &\quad \times\sin(k_3d_3)\sin(k_4d_4). \end{aligned} \tag{A2}$$

(3) Each cell contains six sublayers:

$$\begin{aligned} A + D &= \cos(k_1d_1)\cos(k_2d_2)\cos(k_3d_3)\cos(k_4d_4)\cos(k_5d_5) \\ &\quad \times\cos(k_6d_6) - \frac{1}{2}\left(\frac{z_1}{z_2}+\frac{z_2}{z_1}\right)\sin(k_1d_1)\sin(k_2d_2) \\ &\quad \times\cos(k_3d_3)\cos(k_4d_4)\cos(k_5d_5)\cos(k_6d_6) \\ &\quad -\frac{1}{2}\left(\frac{z_2}{z_3}+\frac{z_3}{z_2}\right)\sin(k_2d_2)\sin(k_3d_3)\cos(k_1d_1)\cos(k_4d_4) \\ &\quad \times\cos(k_5d_5)\cos(k_6d_6) - \frac{1}{2}\left(\frac{z_3}{z_4}+\frac{z_4}{z_3}\right)\sin(k_3d_3) \\ &\quad \times\sin(k_4d_4)\cos(k_1d_1)\cos(k_2d_2)\cos(k_5d_5)\cos(k_6d_6) \\ &\quad -\frac{1}{2}\left(\frac{z_1}{z_3}+\frac{z_3}{z_1}\right)\sin(k_1d_1)\sin(k_3d_3)\cos(k_2d_2)\cos(k_4d_4) \\ &\quad \times\cos(k_5d_5)\cos(k_6d_6) - \frac{1}{2}\left(\frac{z_2}{z_4}+\frac{z_4}{z_2}\right)\sin(k_2d_2) \\ &\quad \times\sin(k_4d_4)\cos(k_1d_1)\cos(k_3d_3)\cos(k_5d_5)\cos(k_6d_6) \\ &\quad -\frac{1}{2}\left(\frac{z_1}{z_4}+\frac{z_4}{z_1}\right)\sin(k_1d_1)\sin(k_4d_4)\cos(k_2d_2)\cos(k_3d_3) \\ &\quad \times\cos(k_5d_5)\cos(k_6d_6) - \frac{1}{2}\left(\frac{z_1}{z_5}+\frac{z_5}{z_1}\right)\sin(k_1d_1) \\ &\quad \times\sin(k_5d_5)\cos(k_2d_2)\cos(k_3d_3)\cos(k_4d_4)\cos(k_6d_6) \\ &\quad -\frac{1}{2}\left(\frac{z_1}{z_6}+\frac{z_6}{z_1}\right)\sin(k_1d_1)\sin(k_6d_6)\cos(k_2d_2)\cos(k_3d_3) \end{aligned}$$

$$
\begin{aligned}
&\times \cos(k_4d_4)\cos(k_5d_5) - \frac{1}{2}\left(\frac{z_2}{z_5}+\frac{z_5}{z_2}\right)\sin(k_2d_2)\\
&\times \sin(k_5d_5)\cos(k_1d_1)\cos(k_3d_3)\cos(k_4d_4)\cos(k_6d_6)\\
&-\frac{1}{2}\left(\frac{z_2}{z_6}+\frac{z_6}{z_2}\right)\sin(k_2d_2)\sin(k_6d_6)\cos(k_1d_1)\cos(k_3d_3)\\
&\times \cos(k_4d_4)\cos(k_5d_5) - \frac{1}{2}\left(\frac{z_3}{z_5}+\frac{z_5}{z_3}\right)\sin(k_3d_3)\\
&\times \sin(k_5d_5)\cos(k_1d_1)\cos(k_2d_2)\cos(k_4d_4)\cos(k_6d_6)\\
&-\frac{1}{2}\left(\frac{z_3}{z_6}+\frac{z_6}{z_3}\right)\sin(k_3d_3)\sin(k_6d_6)\cos(k_1d_1)\cos(k_2d_2)\\
&\times \cos(k_4d_4)\cos(k_5d_5) - \frac{1}{2}\left(\frac{z_4}{z_5}+\frac{z_5}{z_4}\right)\sin(k_4d_4)\\
&\times \sin(k_5d_5)\cos(k_1d_1)\cos(k_2d_2)\cos(k_3d_3)\cos(k_6d_6)\\
&-\frac{1}{2}\left(\frac{z_4}{z_6}+\frac{z_6}{z_4}\right)\sin(k_4d_4)\sin(k_6d_6)\cos(k_1d_1)\cos(k_2d_2)\\
&\times \cos(k_3d_3)\cos(k_5d_5) + \frac{1}{2}\left(\frac{z_5}{z_6}+\frac{z_6}{z_5}\right)\sin(k_5d_5)\\
&\times \sin(k_6d_6)\cos(k_1d_1)\cos(k_2d_2)\cos(k_3d_3)\cos(k_4d_4)\\
&+\frac{1}{2}\left(\frac{z_1z_3}{z_2z_4}+\frac{z_2z_4}{z_1z_3}\right)\sin(k_1d_1)\sin(k_2d_2)\sin(k_3d_3)\\
&\times \sin(k_4d_4)\cos(k_5d_5)\cos(k_6d_6) + \frac{1}{2}\left(\frac{z_1z_3}{z_2z_6}+\frac{z_2z_6}{z_1z_3}\right)\\
&\times \sin(k_1d_1)\sin(k_2d_2)\sin(k_3d_3)\sin(k_6d_6)\cos(k_4d_4)\\
&\times \cos(k_5d_5) + \frac{1}{2}\left(\frac{z_2z_5}{z_1z_4}+\frac{z_1z_4}{z_2z_5}\right)\sin(k_1d_1)\sin(k_2d_2)\\
&\times \sin(k_4d_4)\sin(k_5d_5)\cos(k_3d_3)\cos(k_6d_6)\\
&+\frac{1}{2}\left(\frac{z_3z_5}{z_1z_4}+\frac{z_1z_4}{z_3z_5}\right)\sin(k_1d_1)\sin(k_3d_3)\sin(k_4d_4)\\
&\times \sin(k_5d_5)\cos(k_2d_2)\cos(k_6d_6) + \frac{1}{2}\left(\frac{z_2z_6}{z_1z_4}+\frac{z_1z_4}{z_2z_6}\right)\\
&\times \sin(k_1d_1)\sin(k_2d_2)\sin(k_4d_4)\sin(k_6d_6)\cos(k_3d_3)\\
&\times \cos(k_5d_5) + \frac{1}{2}\left(\frac{z_3z_6}{z_1z_4}+\frac{z_1z_4}{z_3z_6}\right)\sin(k_1d_1)\sin(k_3d_3)\\
&\times \sin(k_4d_4)\sin(k_6d_6)\cos(k_2d_2)\cos(k_5d_5)\\
&+\frac{1}{2}\left(\frac{z_2z_6}{z_1z_5}+\frac{z_1z_5}{z_2z_6}\right)\sin(k_1d_1)\sin(k_2d_2)\sin(k_5d_5)\\
&\times \sin(k_6d_6)\cos(k_3d_3)\cos(k_4d_4) + \frac{1}{2}\left(\frac{z_3z_6}{z_1z_5}+\frac{z_1z_5}{z_3z_6}\right)\\
&\times \sin(k_1d_1)\sin(k_3d_3)\sin(k_5d_5)\sin(k_6d_6)\cos(k_2d_2)\\
&\times \cos(k_4d_4) + \frac{1}{2}\left(\frac{z_4z_6}{z_1z_5}+\frac{z_1z_5}{z_4z_6}\right)\sin(k_1d_1)\sin(k_4d_4)\\
&\times \sin(k_5d_5)\sin(k_6d_6)\cos(k_2d_2)\cos(k_3d_3)\\
&+\frac{1}{2}\left(\frac{z_3z_5}{z_2z_4}+\frac{z_2z_4}{z_3z_5}\right)\sin(k_2d_2)\sin(k_3d_3)\sin(k_4d_4)\\
&\times \sin(k_5d_5)\cos(k_1d_1)\cos(k_6d_6) + \frac{1}{2}\left(\frac{z_3z_6}{z_2z_4}+\frac{z_2z_4}{z_3z_6}\right)\\
&\times \sin(k_2d_2)\sin(k_3d_3)\sin(k_4d_4)\sin(k_6d_6)\cos(k_1d_1)\\
&\times \cos(k_5d_5) + \frac{1}{2}\left(\frac{z_3z_6}{z_2z_5}+\frac{z_2z_5}{z_3z_6}\right)\sin(k_2d_2)\sin(k_3d_3)\\
&\times \sin(k_5d_5)\sin(k_6d_6)\cos(k_1d_1)\cos(k_4d_4)\\
&+\frac{1}{2}\left(\frac{z_4z_6}{z_2z_5}+\frac{z_2z_5}{z_4z_6}\right)\sin(k_2d_2)\sin(k_4d_4)\sin(k_5d_5)\\
&\times \sin(k_6d_6)\cos(k_1d_1)\cos(k_3d_3) + \frac{1}{2}\left(\frac{z_4z_6}{z_3z_5}+\frac{z_3z_5}{z_4z_6}\right)\\
&\times \sin(k_3d_3)\sin(k_4d_4)\sin(k_5d_5)\sin(k_6d_6)\cos(k_1d_1)\\
&\times \cos(k_2d_2) - \frac{1}{2}\left(\frac{z_1z_3z_5}{z_2z_4z_6}+\frac{z_2z_4z_6}{z_1z_3z_5}\right)\sin(k_1d_1)\\
&\times \sin(k_2d_2)\sin(k_3d_3)\sin(k_4d_4)\sin(k_5d_5)\sin(k_6d_6).
\end{aligned}
\tag{A3}
$$

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