Analysis of shear modes in a piezoelectric vibrator

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A shear piezoelectric vibrator has been analyzed in detail. There are two types of shear motion associated with the same shear stain deformation. It is found experimentally that the lower frequency mode among the two has lower strength and its vibration amplitude continuously decreases as the long dimension of the vibrator increases. A nearly pure shear mode may be obtained with the aspect ratio greater than 20, but the influence of the other low frequency mode could not be totally eliminated even at such a large aspect ratio. We found that such phenomena can be understood by the conservation of angular momentum and the shared origin of these two types of shear motion. The ratio of the driving forces for these two types of shear motion is related directly to the aspect ratio of the vibrator. Formulas for calculating k_{15} from the resonant and antiresonant frequencies using both type of shear modes are given. © *1998 American Institute of Physics.* [S0021-8979(98)02908-9]

I. INTRODUCTION

Piezoelectric materials have been widely used in various ultrasonic devices. One of the basic parameters used to describe the quality of piezoelectric materials is the electromechanical coupling factor, which is a measure of the effectiveness of electromechanical energy conversion. Accurately determining the shear coupling coefficient k_{15} is of practical importance for making shear transducers and for parameter input in theoretical modeling. It can also affect the accuracy of many derived material properties for a piezoelectric material.

In piezoceramics, such as lead zirconate-titanate (PZT), the symmetry is ∞m after poling, the shear coupling coefficient k_{15} can be related to the following material constants:

$$\epsilon_{11}^S = \epsilon_{11}^T (1 - k_{15}^2), \tag{1a}$$

$$d_{15} = k_{15}\sqrt{\epsilon_{11}^T s_{55}^E}, \tag{1b}$$

$$c_{55}^E = c_{55}^D (1 - k_{15}^2), \tag{1c}$$

where ϵ_{11}^S and ϵ_{11}^T are the clamped and free dielectric permittivities perpendicular to the poling direction, respectively, d_{15} is the shear piezoelectric constant, c_{55}^E and c_{55}^D are shear elastic stiffness constants under constant electric field and constant electric displacement, respectively.

This shear coupling coefficient k_{15} is usually measured using the thickness-shear resonance following the Institute of Electrical and Electronic Engineers (IEEE) standards.[1,2] However, the measurement cannot be easily made with high accuracy due to the coupling of the thickness-shear mode with other unwanted modes. The coupling effect is usually more pronounced for the antiresonant frequency. For this reason, the method of using electrical resonance and the antiresonance to determine k_{15} is not recommended by the IRE and IEEE standards.[1,3] Instead, a dielectric measurement method was given according to Eq. (1a).[3,4] It is easy to obtain the free permittivity ϵ_{11}^T value by measuring the capacitance at a low frequency (<1 kHz) which is well below the fundamental resonance, however, the clamped permittivity ϵ_{11}^S at very high frequencies is not easily accessible because of the influence of those mechanical resonances and/or their higher harmonics. Moreover, many piezoelectric ceramics also have fairly large capacitance relaxation which affects high frequency measurements.[5]

An alternative method to determine k_{15} is to measure values of the fundamental and overtone resonant frequencies instead of the fundamental resonant and antiresonant frequencies.[6,7] The idea is that the unwanted modes have lower frequencies so that the influence becomes smaller for the overtones at high frequencies. Although this method is more convenient, experiments show that it gives lower values than those obtained by other techniques. In addition, if the aspect ratio of the vibrator is not large enough, the coupling effect will be extended to the overtones of the thickness-shear mode making this method invalid. Up to now, the reason for this shear mode to have such an unusual strong coupling was not fully understood.

More interestingly, the thickness-shear is *not* the lowest resonance of the structure recommended for the resonance technique, yet has the largest amplitude, which itself is a puzzle. Generally speaking, in most resonance structures, the lowest resonance usually has the largest amplitude. Our goal here is to find out the nature of this puzzle and to define quantitatively the required dimensions of the vibrator to perform a dependable measurement.

It is well known that the impedance-frequency characteristic of a vibrator used for determining k_{15} strongly depends on its geometry. However, the recommended ratios among the length l, width w, and thickness t are inconsistent in the literature. Experimentally, we could not obtain satisfactory results using some of those recommended dimensions.

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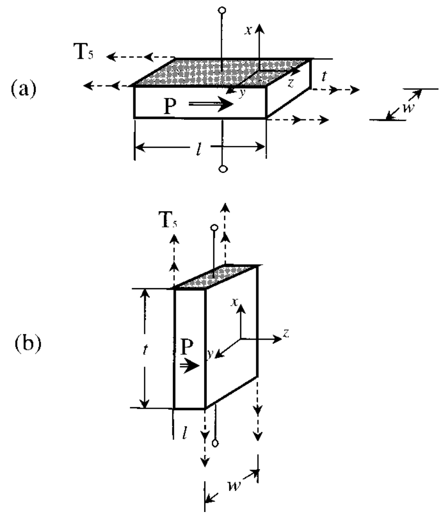

FIG. 1. (a) Thickness-shear vibrator. (b) Length-shear vibrator. The shaded areas are the electrodes and the dashed arrows represent the direction of displacement at given points. The polarization direction is indicated by an arrow on the front face of the sample.

Through detailed analysis, we have established the optimized dimensions for which the k_{15} value can be accurately determined using the resonance technique. More importantly, we have given an explanation to the cause of this strong mode coupling and the explained the amplitude relationship.

II. EXPERIMENTAL ANALYSIS OF THICKNESS-SHEAR VIBRATOR

For a thickness-shear mode piezoelectric ceramic vibrator under free boundary conditions, the following equation[8,9] holds,

$$\tan \Xi = \frac{\Xi}{k_{15}^2}, \tag{2}$$

where $\Xi = \pi f_{nr} t / v^D$ is a normalized frequency, f_{nr} is the resonant frequency of the nth mode, t is the thickness of the vibrator, $v^D = \sqrt{c_{55}^D / p}$ is the stiffened shear velocity of the elastic wave. The ratio of an overtone to fundamental frequency f_{nr}/f_r versus value of k_{15} was tabulated in Ref. 6 according to Eq. (2). In principle, one could directly obtain the values of k_{15} for any given f_{nr}/f_r ratio using this table.

For a pure mode, Eq. (2) can also be converted to

$$k_{15} = \sqrt{\frac{\pi f_r}{2 f_a} \cot \frac{\pi f_r}{2 f_a}} = \sqrt{\frac{\pi f_r}{2 f_a} \tan \frac{\pi \Delta f}{2 f_a}}, \tag{3}$$

where f_r, f_a are the fundamental resonant and antiresonant frequencies, $\Delta f = f_a - f_r$ is the resonance bandwidth. Therefore, k_{15} may also be evaluated by measuring the fundamental resonant and antiresonant frequencies from the impedance spectrum if the mode can be decoupled from others.

In our experiments, the thickness-shear mode samples are made of PZT-5H piezoelectric ceramic purchased from Morgan Matroc Inc. The typical shape of the vibrators is a rectangular bar as shown in Fig. 1(a). For a thickness-shear mode vibrator, the polarization direction is along the longest dimension represented by l. The two major surfaces (top and bottom), which are parallel to each other and to the direction of polarization, were coated with gold or silver electrodes. When an AC voltage is applied across the electrodes, shear mechanical vibration can be excited in the vibrator through piezoelectric effect. Several samples with different dimensions were designed and fabricated. Considering that the dimensions, l and w, of the electrode surface are sufficiently larger than the thickness t and the relatively large permittivity of PZT-5H, edge effects of the parallel plate capacitor can be neglected in the computation of the internal electric field.

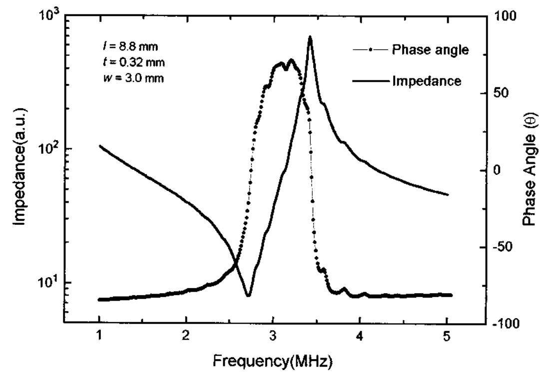

FIG. 2. Impedance-frequency spectrum of a typical thickness-shear vibrator.

A HP-4194A impedance/gain-phase analyzer with computer interface was used for the impedance-frequency spectrum measurements. The electric field used was relatively low, typically in the range of 0.05–0.2 V/mm. The PZT samples are perfectly linear in this field range.

The impedance amplitude and phase spectra for a typical thickness-shear mode vibrator are shown in Fig. 2. This vibrator has the following dimensions: $l = 8.8$ mm, $w = 3.0$ mm, and $t = 0.32$ mm. The frequencies of the fundamental resonance and antiresonance are 2.70 MHz and 3.41 MHz, respectively. Using Eq. (3), one can easily calculate the shear electromechanical coupling coefficient to be: $k_{15} = 0.650$.

In all other vibrators used for measuring material parameters, such as the plates and bars for the measurement of k_t, k_{33}, etc., mode coupling could be eliminated as long as the dimension corresponding to the desired fundamental mode is larger than four times of the other dimensions. It is very difficult, however, to obtain a clean thickness-shear mode. We found that the thickness-shear mode suffers mode coupling even when the aspect ratio l/t is more than 10. In order to find the origin of such an unusual behavior, a series of samples with the same thickness but different length were made and measured. Figure 3 shows the comparison of the impedance-frequency spectra of these samples near the thickness-shear resonance. The same spectra with wider frequency region are shown in Fig. 4 to demonstrate the overtone frequencies. Based on these measurements, k_{15} was calculated and the results are summarized in Table I.

Figures 3 and 4 show that the impedance-frequency spectra of the thickness-shear mode depends strongly on the ratio of l/t but nearly independent of the ratio l/w. Since these measured vibrator samples have identical thickness, the

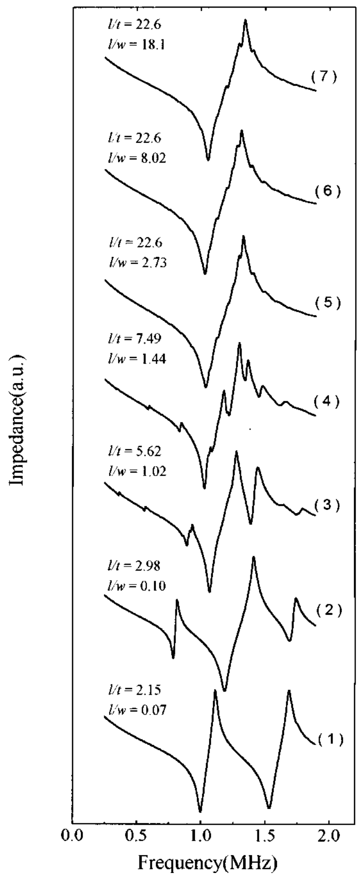

FIG. 3. Impedance-frequency characteristics near the fundamental resonance and antiresonance for thickness-shear vibrators of different aspect ratio.

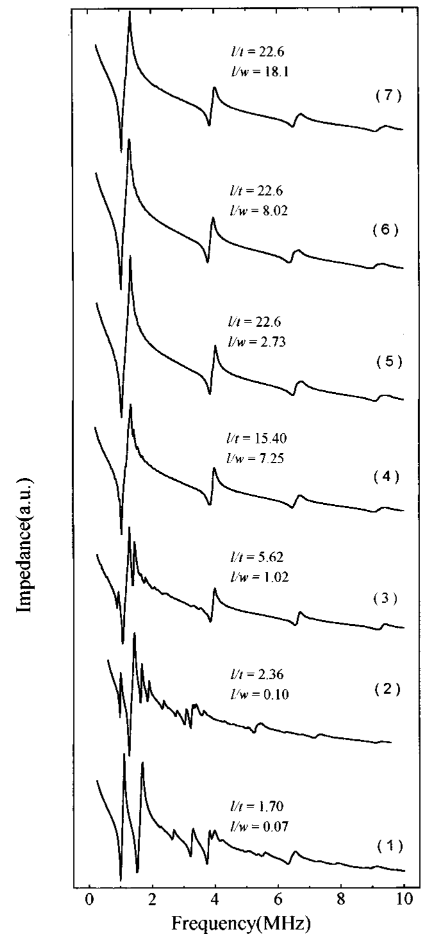

FIG. 4. Overtones for thickness-shear vibrators of different aspect ratio.

above results imply that the interference mode is related to vibrator's length only. For a length to thickness ratio $l/t > 20$, although the mode is still not pure, the resonance and antiresonance frequencies can be accurately defined without any ambiguity. For the aspect ratio $l/t = 10-20$, coupling effects appear on the spectrum of the thickness-shear mode (see curve 4 in Fig. 3). However, the peaks are recognizable and k_{15} may still be determined from the minimum and maximum of the impedance curve without too much error. The mode coupling gets stronger as the ratio l/t decreases, no clearly defined thickness-shear mode can be found when the ratio $l/t < 5$. Apparently, this shear vibrator is much more demanding compared to the other types of vibrators for measuring k_t and k_{33}, for which an aspect ratio of 4 is already sufficient to separate the modes.

In Fig. 3, curves 5–7 are for samples with identical length l and thickness t, but different width w. The three curves show that the width of the vibrator has little influence on the resonant and antiresonant frequencies if the l/t ratio is large enough. Once again, our experimental results point to the origin of the interference mode to be solely related to the sample length.

From Eq. (1c) the shear coupling coefficient k_{15} may also be determined from the ratio of the shear elastic constants under constant electric field and constant electric displacement, respectively. These elastic constants could be measured by using ultrasonic technique.[10,11] Using the same set of samples, we have measured the shear velocity with the displacement along and perpendicular to the poling direction (the wave vector is always perpendicular to the polarization), which give rise to the values for c_{55}^E and c_{55}^D, respectively. Since the ultrasonic measurements were performed at nonresonance conditions, there are no mode interference problems. The k_{15} value so obtained is geometry independent as long as the sample dimensions are not too small compared to the wavelength of the ultrasonic signal and the degree of poling is maintained the same. We have also developed a signal processing technique for measuring subwavelength thick samples so that accurate measurements on the elastic stiffness constants can be performed even for very small and thin samples.[12] The k_{15} value obtained by ultrasonic method is used as the standard to analyze the other results listed in Table I. It is found that for large l/t ratio samples, the k_{15} values obtained from the resonant and antiresonant frequencies are very close to the one obtained from the ultrasonic measurement. However, the k_{15} values calculated by using the ratio of overtone to fundamental frequencies are slightly smaller, which may due to the effects of frequency dispersion of the materials properties.

III. LENGTH-SHEAR MODE

From the above experimental analysis, the mode interference is related to the length dimension l. Looking more

TABLE I. Comparison of measured k_{15} values from different samples and different techniques.

Sample	Length l (mm)	Width w (mm)	Thickness t (mm)	$l:t$	$l:w$	Resonance f_r (MHz)	Anti-resonance f_a (MHz)	k_{15} (f_r/f_a)	k_{15} (Overtone)
1	1.43	20.17	0.84	1.70	0.07	1.54300	1.6880	0.441	
2	1.98	20.17	0.84	2.36	0.10	1.18300	1.4090	0.582	0.492
3	3.30	20.17	0.84	3.92	0.16	1.10790	1.3770	0.633	0.541
4	4.72	4.63	0.84	5.62	1.02	1.06780	1.2756	0.587	0.626
5	6.16	20.17	0.84	7.33	0.31	1.05210	1.2330	0.560	0.640
6	6.67	4.64	0.84	7.94	1.44	1.02550	1.2985	0.651	0.621
7	12.65	1.75	0.84	15.06	7.23	1.03157	1.3240	0.665	0.635
8	19.02	6.96	0.84	22.6	2.73	1.03530	1.3278	0.664	0.650
9	19.02	3.50	0.84	22.6	5.43	1.02875	1.3213	0.656	0.646
10	19.02	2.37	0.84	22.6	8.02	1.02875	1.3115	0.659	0.649
11	19.02	1.75	0.84	22.6	10.9	1.02875	1.3278	0.670	0.642
12	19.02	1.05	0.84	22.6	18.1	1.02963	1.3089	0.656	0.629
Ultrasonic	6.62	6.42	6.35					0.665	

carefully, one can find that another type of shear motion is also excited, which we call it the length-shear in contrast to the thickness-shear. In this mode, the displacement is perpendicular to the poling direction while in the thickness-shear, the displacement is parallel to the poling direction.

For a piezoelectric material, the origin of the driving force for the shear motion is an electric field induced shear strain, S_5, which is related to the displacement components, u_1 and u_3, in the following form:

$$S_5=\frac{1}{2}\left(\frac{\partial u_1}{dz}+\frac{\partial u_3}{dx}\right). \tag{4}$$

For the thickness-shear motion, the displacement u_3 is assumed to be zero for a long-bar shaped vibrator as shown in Fig. (1a). In this case, the shear strain is given by the first term of Eq. (4) only, and the resonance could be derived to have the form given by Eqs. (2) and (3). On the other hand, the length-shear mode, which gives rise to the same shear strain was ignored in the derivation of the formula. Due to the symmetric nature of the two shear displacement derivatives in Eq. (4), we could imagine a situation in which the thickness-shear mode may be ignored if the dimensions are designed properly. The impedance formula for this mode can be easily derived if we assume the displacement $u_1=0$. In this case, the shear strain is only related to the second term of Eq. (4). Using the dynamical equation and the following constitutive relations:

$$S_5=s_{55}^E T_5+d_{15}E_1, \tag{5a}$$

$$D_1=d_{15}T_5+\epsilon_{11}^T E_1, \tag{5b}$$

one can derive another relationship parallel to Eq. (2) for the length-shear mode:

$$\tan\,\Xi'=\left(\frac{k_{15}^2-1}{k_{15}^2}\right)\Xi', \tag{6}$$

where $\Xi'=(\pi f'_{\mathrm{na}}l)/v^E$ is a normalized frequency, f'_{na} is the antiresonant frequency in the nth mode, l is the length of the vibrator, $v^E=1/\sqrt{ps_{55}^E}$ is the compliance velocity of the elastic wave. In Eq. (5), s_{55}^E is the shear elastic compliance at constant electric field, d_{15} is the piezoelectric strain constant, and ϵ_{11}^T is the dielectric permittivity at constant stress.

Using Eq. (6) and considering the fundamental resonance and antiresonance, we can get another k_{15} formula for the length-shear mode:

$$k_{15}=\frac{1}{\sqrt{1-\dfrac{\tan\left(\dfrac{\pi}{2}\dfrac{f'_a}{f'_r}\right)}{\dfrac{\pi}{2}\dfrac{f'_a}{f'_r}}}}, \tag{7}$$

where f'_a, f'_r are the fundamental resonant and antiresonant frequencies of the length-shear mode.

In order to verify this analysis we have made a sequence of samples with the $l:t$ ratio from 1:22 to 22:1. Here l is always the dimension along the poling direction although it may become the shorter one. Figure 5 shows the dependence of the impedance on frequency for two different aspect ratio vibrators. A nearly pure length-shear mode can be obtained when $l/t<1/20$, which is analogous to the thickness-shear case of $l/t>20$. The curves in Fig. 5(b) correspond to the ratio of $t/l=27$. The shear coupling coefficient k_{15} ($=0.650$) can be obtained using Eq. (7), which is the same as the k_{15} values measured by the nearly pure thickness-shear mode and ultrasonic method (see Table I). From this study, we concluded that the tangled modes in the shear vibrator are the thickness-shear and the length-shear modes. The fundamental reason of why it is so difficult to get rid of the mode coupling in the shear resonator is because the two shear modes share the same electric filed induced shear strain. The higher harmonics of the lower frequency mode appear to decorate the higher frequency mode on the impedance spectrum.

IV. CONSERVATION OF ANGULAR MOMENTUM

For the recommended geometry in the literature, the shear vibrator is usually made with the $l/t\gg1$. Therefore, the

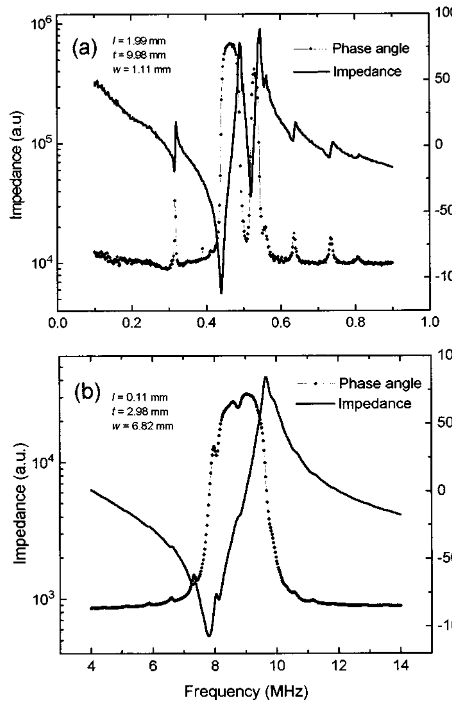

FIG. 5. Impedance spectrum of a length-shear vibrator with different geometry. (a) The ratio of $l/t=0.2$. (b) The ratio of $l/t=0.04$.

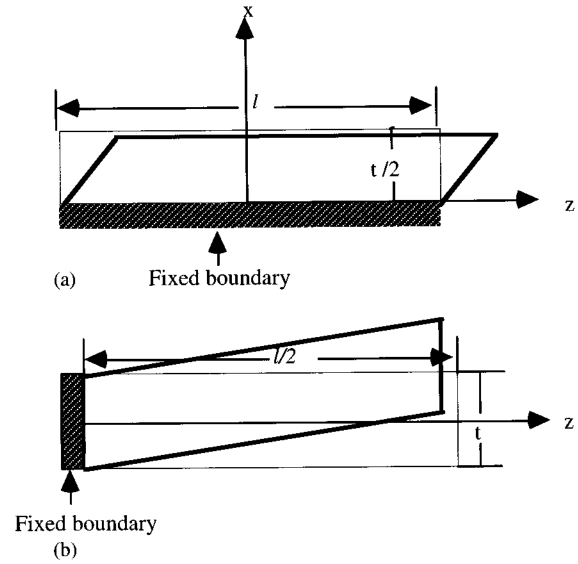

FIG. 6. Illustration of the two shear modes: (a) Thickness shear; (b) length shear. The angular momentum is not conserved in either case.

length-shear mode should be the lowest frequency mode according to the analysis above. However, experimental results show that the shear mode in the long dimension is much weaker than the shear mode in the shorter dimension. This fact is also true for the case of $l/t \ll 1$ as shown in Fig. 5. In other words, the higher frequency mode is much stronger than the lower frequency mode in the shear vibrator. This result is unusual since in most of the cases, lower frequency mode is stronger than higher frequency ones. There is no explanation in the literature for this unusual phenomenon.

Through careful analysis, we conclude that the origin of such a behavior comes from the fact that the shear deformation involves rotation but the electric drive does not provide angular moment since it is due to self-induced shear strain. Therefore, both shear motions should be involved to balance the angular momentum of the whole vibrator.

Let us look at the two special situations shown in Figs. 6(a) and 6(b) assuming only one of the modes is being excited. Figure 6(a) represents the thickness-shear mode for which the center plane along the thickness direction remains still according to symmetry. Therefore, we may analyze the upper half of the vibrator and assume a fixed boundary at the bottom. Similarly, we can analyze the length-shear mode as shown in Fig. 6(b). According to Eq. (4), the two shear displacements are connected to the same shear deformation S_5 which was driven by the external electric field through piezoelectric effect. It is clear that both types of shear motions involve rotation and hence, the angular momentum will not be conserved if only one type of shear motion is excited. This is the intrinsic reason for the strong mode coupling observed in the shear vibrators.

According to the symmetry of the vibrator, to conserve the angular momentum of the system, the shear deformation should proceed to expand along one of the diagonal direction and contract along the other as shown in Fig. 7. Therefore, the ratio of the maximum displacements for the two types of shear motion can be easily found to be

$$\frac{u_{1\,\max}}{u_{3\,\max}} = \frac{l}{t}. \tag{8}$$

Equation (8) can also be used to derive the ratio of the mechanical driving force for the two types of shear motion. There are three conclusions that can be drawn from the above analysis.

(1) The mode corresponding to the shear motion along the longer dimension is always stronger than the one along the shorter dimension. The amplitude ratio between the displacements in the two directions is proportional to the ratio of their corresponding dimensions.
(2) The two types of shear displacements will always occur in a paired fashion to conserve the angular momentum. However, the lower frequency mode can be weakened through the increase of the longer dimension.
(3) These two shear motions are equivalent, their relative strength can be adjusted by the aspect ratio of the two relevant dimensions. Either mode could be used to measure the material properties by proper design of the vibrator dimensions to suppress the other one.

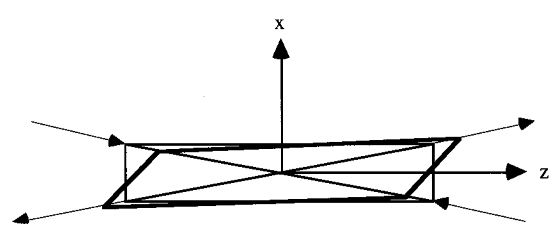

FIG. 7. The shear deformation which conserves the angular momentum. The two diagonal lines have no rotation during the shear deformation and the ratio of the two shear displacements is l/t.

The first conclusion explains the experimentally observed phenomena of weaker lower frequency mode and stronger high frequency mode because the amplitude of motion reflects the strength of the driving force, while the second conclusion can be verified from the results shown in Figs. 3 and 5. The frequency spectrum is still not smooth in the vicinity of the desired resonance even for an aspect ratio of larger than 22. The third conclusion has been verified experimentally as shown in Fig. 5(b) and Table I.

V. CONCLUSION

Our experimental results and theoretical analysis proved that the interference in the thickness-shear vibrator comes from the length-shear mode. The frequency of the length-shear mode depends on the length while its intensity is controlled by the length to thickness ratio l/t. With the increase of the l/t ratio, the length-shear mode can be weakened but could not be eliminated. Experimentally, we found that when $l/t \geqslant 20$, a relatively clean thickness-shear mode may be obtained. Therefore, this ratio should be used to define the dimensions if one wants to use the resonance technique to measure k_{15}. In the aspect ratio range of $10 < l/t < 20$, the impedance-frequency spectrum near the thickness-shear mode is slightly obscured by the length-shear mode. However, the resonant and antiresonant frequencies of the thickness-shear mode are still clearly shown on the spectrum and the k_{15} value calculated from the minimum and the maximum frequencies is still acceptable. When the ratio $l/t < 10$, the resonance method is not valid to determine k_{15} (see Table I).

From our experimental results, the width variation of the vibrator has no obvious influence to the spectrum of the thickness-shear mode. The effect of changing the width is only on the total capacitance which may have some influence on the precision of the effective digit in the measured values. It is recommended that the width should be at least twice as much as the thickness to reduce the edge effect of the plate capacitor.

The length-shear mode is a ''twin'' of the thickness-shear mode since they both share the same shear strain. We found that k_{15} may also be measured accurately using this mode if the aspect ratio $t/l > 20$, which mirrors the situation for the thickness-shear mode. However, owing to the fact that the electrodes of the length-shear vibrator are coated on a pair of minor surfaces and the relatively large distance between the two electrodes, the measurement is more difficult than using the thickness-shear mode. The plate capacitance approximation is less accurate for the length-shear vibrator due to some degree of edge effects. This, however, was not a problem for the PZT ceramic samples we used in our experiment since the relative dielectric constant is over 1300.

Using the idea of angular momentum conservation, we have shown that the ratio of the driving forces for the thickness-shear and the length-shear is roughly proportional to l/t. This explains the fact that the lower frequency mode has much smaller amplitude than that of the high frequency one. One could arbitrarily reduce the mode interference by increasing the aspect ratio, but the influence of the lower frequency mode will always exist due to the same excitation source and the conservation of angular momentum.

ACKNOWLEDGMENTS

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