

Math 411 - Ordinary Differential Equations

- The second Midterm Exam will take place on Wednesday November 18, 6:30–7:45 pm, Osmond 109.

Practice Exam II

1. Show that $V(x, y) = \frac{x^4}{2} + y^2$ is a Lyapunov function for the planar system

$$\begin{cases} x' = y \\ y' = -x^3 - y \end{cases}$$

Prove that the origin is an asymptotically stable equilibrium.

2. Determine for which values of the constant α the system

$$\begin{cases} x' = \alpha \sin x - y \\ y' = e^x - \cos y \end{cases}$$

has an asymptotically stable equilibrium at the origin.

3. Sketch the null-clines and draw the phase plane diagram for the linear system

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Classify this system (center, spiral, saddle, node, stable, unstable...)

4. Prove that the following systems have no cycles:

$$\begin{cases} x' = x - y \\ y' = 2 + xy^3 \end{cases} \quad \begin{cases} x' = x^3 + 2xy + \sin y \\ y' = \cos x + y(1 - y) \end{cases}$$

5. Show that

$$\begin{cases} x' = x + y + 4y^3 \\ y' = -(x + y) \end{cases}$$

is a hamiltonian system. Show that this system has only one equilibrium point, but infinitely many cycles.