MATH 597E: Problem Set 1
due Wednesday, February 16, 2011

1. Dorlas: Problems 1, 2, 4 - p.84

2. For the following, use the energy and equation of state relations for an ideal gas as discussed in class

   (a) Prove that

   \[ p = \frac{2U}{3V} \]

   where \( p \) is the pressure, \( V \) is the volume, and \( U \) is the total energy. In other words, the pressure of an ideal gas is proportional to its energy density (energy per volume). Does this relation depend on any properties of the ideal gas?

   (b) Use this to prove that \( UV^{2/3} = \text{const} \) for an adiabatic process.

   (c) Derive the “Law of Partial Pressures”: assume that \( n_1 \) moles of one type of gas and \( n_2 \) moles of another type of gas are contained in the same volume \( V \) (mixed) at temperature \( T \). Show that the pressure \( p = p_1 + p_2 \), where \( p_k \) is the pressure that the \( k \)th gas would have if it occupied the volume alone.

3. Consider the following nonlinear version of the experimental facts/mathematical axioms for an ideal gas: \( U = aT^m \), \( pV = bT^n \), where \( a \) and \( b \) are constants, and \( n, m \) are parameters. The case of an ideal gas corresponds to \( n = 1 \) (gas law) and \( m = 1 \) (Joule energy for a gas). Rederive the relationship between the heat capacities \( C_p \) and \( C_V \) for arbitrary \( n \) and \( m \), assuming that the 1st Law is unchanged. Can you find any restrictions on \( n \) and \( m \)?

4. Consider a thermodynamic magnetic system defined by the First Law

   \[ dU = dQ + \mu_0 H dM, \]

   where \( M \) is the magnetization of the system (an internal variable, like the pressure), and \( H \) is the magnetic field (externally-controlled, like the volume); \( \mu_0 H dM \) is the magnetic work, with \( \mu_0 \) the fundamental (magnetic) constant. For this system the equation of state is \( M = AH/T \), where \( A \) is a constant.

   By analogy with \( C_p \) and \( C_V \) for ideal gases, use the First Law to define two heat capacities, and derive a relationship between them. Can you define a magnetic quantity like the enthalpy?